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Pe anent and Transitory Fa ors Affecting the Dy mics of the Term St ture of Interest Rates

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Permanent and Transitory Factors Affecting the Dynamics of the Term Structure of Interest Rates*

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Abstract

This paper proposes a novel methodology, based on the Common Principal Component analysis, allowing one to estimate the factors driving the term structure of interest rates, in the presence of time-varying covariance structure. The advantages of this method are first, that, unlike classical principal component analysis, common factors can be estimated without assuming that the volatility of the factors is constant; and second, that the factor structure can be decomposed into permanent and transitory common factors. We conclude that only permanent factors are relevant for modeling the dynamics of interest rates, and that the common principal component approach appears to be more accurate than the classical principal component one to estimate the risk factor structure.

Keywords. Term Structure of Interest Rates; Principal Component Analysis; Common Principal Component Analysis

Executive Summary

This paper proposes a novel methodology, based on the Common Principal Component analysis, allowing one to estimate the factors driving the term structure of interest rates, in the presence of time-varying covariance structure.

The main advantages of the CPC framework can be presented as follows. First, unlike in the classical principal component analysis, the covariance matrix is not supposed to be constant over the considered period, an unrealistic assumption in the case of bond yields. On the other hand, the CPC approach allows the covariance matrix to change from subperiod to subperiod, while still estimating a single common factor structure over the whole sample period. Second, both permanent and transitory, or subperiod-specific, factors can be estimated. In this paper, a factor is said to be permanent if it has the same financial meaning, captured by the factor loadings (eigenvectors), over the whole time period.

Since our methodology is more flexible than the principal component analysis in the case of time-varying covariance matrix structure, it has the potential to advantageously replace it in many financial applications. First, it could be relevant in certain aspects of risk management. For instance, immunization strategies, durations and Value-at-Risk computations, and the reduction in dimension for scenario simulation can be achieved by decomposing the covariance matrix into principal components. Second, the interest-rate derivative literature may also benefit from the framework proposed in this paper. Recently, Driessen, Klaassen and Melenberg (2002) have priced and hedged caps and swaptions, employing successively the Heath, Jarrow and Morton (1992) and the Libor market models, using principal component analysis to estimate the volatility functions. In a closely related paper, Fan, Gupta and Ritchken (2001) investigate the performance of Gaussian, proportional and square-root multi-factor models also basing their estimation procedure on principal

component analysis. Longstaff, Santa-Clara and Schwartz (2001a) study the relative pricing of European-style caps and swap options, and Longstaff, Santa-Clara and Schwartz (2001b) quantify the cost of using a mispecified model of the term structure to price American swap options. In the latter two papers, the authors use a principal component analysis of the historical covariance matrix to estimate the pricing factors, and make the identification assumption that these pricing factors generate also the covariance matrix implied by interest rate derivative prices. Our methodology could be very useful to calibrate all these interest rate derivative pricing models.

The main contribution of the present paper is to propose a new methodology allowing one to estimate the permanent and transitory factors driving the term structure of interest rates. This methodology is based on the CPC model, which is an extension of the classical principal component analysis in the case of several groups. In this paper, we associate for the first time the groups to successive time periods. By initially running a separate principal component analysis on each subperiod, we observe that the factor loadings remain fairly constant across subperiods whereas the volatility of the factors fluctuate extensively through time. These results stay valid regardless of the number and the nature (non-overlapping vs. overlapping and equal size vs. unequal size) of subperiods considered. We also notice that the variance accounted for by the first factors changes substantially from subperiod to subperiod. We then propose different analyses allowing one to estimate either only permanent factors, or both permanent and transitory factors, using successively two, three, four and eight non-overlapping subperiods. We conclude that the factor structure has not changed appreciably and that permanent factors should be estimated using the common principal component approach.

Introduction

Understanding the dynamics of interest rates is crucial for the purpose of managing interest-rate risk exposure and pricing interest-rate derivatives. Comovements among interest rates of different maturities are often summarized by the covariance matrix of bond yields, from which risk factors are often extracted through principal component analysis (see Litterman and Scheinkman, 1991).[1] To the extent that economic and political conditions do change over time, one would expect both the covariance between bond yields and the variance of the risk factors to change as well. The changing nature of this covariance matrix is consistent with empirical evidence (see for instance Engle, Ng and Rothschild, 1990). Moreover, Collin-Dufresne and Goldstein (2001) suggest that the relative mispricing of caps and swaptions, reported by Longstaff, Santa-Clara and Schwartz (2001a), may be attributed to variability in the correlation structure of bond yields over time.

Comparing covariance structure on several time periods is a useful exercise in finance since it allows one to measure the persistence of underlying economic forces over time. Different similarity levels are considered in this paper: no relation between the covariance matrices (all factors are transitory ones); one permanent factor, the others being transitory; two permanent factors; etc; a permanent factor structure; and finally covariance matrix equality. The last assumption requires both a permanent factor structure and a constant variance explained by each factor. This factor decomposition can be achieved

[1] A principal component analysis is a rotation of axes in multidimensional space which allows one to find linear combinations - the principal components - of the original variables that summarize as much of the information as possible. The interpretation of principal components depends on two attributes: the eigenvalue and the eigenvector of each component. The eigenvalue is an estimate of the amount of total variance explained by that particular component. The eigenvector associated with a component is a vector containing the factor loadings, i.e. the weights of the original variables, or their correlation with the given component. Inspecting the eigenvalues allows one to pick the minimum number of components summarizing enough of the total variance, while inspecting the eigenvectors leads to financial interpretations of the principal components.

using the Common Principal Component (CPC) model and its offspring the partial CPC (pCPC) models, proposed by Flury (1984, 1986, 1987, 1988). These models extend the classical principal component analysis in the case of several groups.

The main advantages of the CPC framework can be presented as follows. First, unlike in the classical principal component analysis, the covariance matrix is not supposed to be constant over the considered period, an unrealistic assumption in the case of bond yields. On the other hand, the CPC approach allows the covariance matrix to change from subperiod to subperiod, while still estimating a single common factor structure over the whole sample period. Second, both permanent and transitory, or subperiod-specific, factors can be estimated. In this paper, a factor is said to be permanent if it has the same financial meaning, captured by the factor loadings (eigenvectors), over the whole time period.

Beside these theoretical arguments, the CPC framework is also supported by empirical evidence. Indeed, without any formal tests, Bliss (1997), Phoa (2000), and Chapman and Pearson (2001) have shown that the factor decomposition of the U.S. term structure is robust through time. For instance, Bliss (1997) divides his sample period from 1970 through 1995 into three subperiods. The explanatory power of the factors is calculated over the entire period and over each of the three subperiods. He noticed that although the volatility of the factors (eigenvalues) vary across subperiods, the factor loadings show a consistent pattern across the different subperiods. As it appears empirically that the eigenvectors remain fairly constant across subperiods but the volatility of the factors fluctuate over time, the CPC model is believed to be particularly accurate for modeling the dynamics of the bond yields.

Since our methodology is more flexible than the principal component analysis in the case of time-varying covariance matrix structure, it has the potential to advantageously replace it in many financial applications. First, it could be relevant in certain aspects of risk management. For instance, immunization strategies (Barber and Copper, 1996), durations (Litterman and Scheinkman, 1991) and Value-at-Risk computations (Singh, 1997), and the reduction in dimension for scenario simulation (Jamshidian and Zhu, 1996) can be achieved by decomposing the covariance matrix into principal components. Second, the interest-rate derivative literature may also benefit from the framework proposed in this paper. Recently, Driessen, Klaassen and Melenberg (2002) have priced and hedged caps and swaptions, employing successively the Heath, Jarrow and Morton (1992) and the Libor market models, using principal component analysis to estimate the volatility functions. In a closely related paper, Fan, Gupta and Ritchken (2001) investigate the performance of Gaussian, proportional and square-root multi-factor models also basing their estimation procedure on principal component analysis. Using the string-shock framework of Goldstein (2000) and Santa-Clara and Sornette (2001), Longstaff, Santa-Clara and Schwartz (2001a) study the relative pricing of European-style caps and swap options, and Longstaff, Santa-Clara and Schwartz (2001b) quantify the cost of using a mispecified model of the term structure to price American swap options. In the latter two papers, the authors use a principal component analysis of the historical covariance matrix to estimate the pricing factors, and make the identification assumption that these pricing factors generate also the covariance matrix implied by interest rate derivative prices. Our methodology could be very useful to calibrate all these interest rate derivative pricing models.

Moreover, since our framework allows for time-varying covariance matrices, it is directly related to previous papers dealing with parsimonious heteroscedastic models. First, Engle, Ng and Rothschild (1990) suggest to use the Factor-ARCH specification to estimate a parsimonious structure for the conditional covariance matrix of asset excess-returns. In their model, the instantaneous covariance matrices are joint diagonalized by a single orthogonal matrix. Second, in his stochastic covariance string market model, Han (2001) assumes that the historical covariance matrix and all the instantaneous implied covariance matrices are diagonalized by the same orthogonal matrix. As this joint diagonalization is the cornerstone hypothesis of the CPC model, our methodology is perfectly consistent with, and our results strongly confirm, the latter two specifications.

The main contribution of the present paper is to propose a new methodology allowing one to estimate the permanent and transitory factors driving the term structure of interest rates. This methodology is based on the CPC model, which is an extension of the classical principal component analysis in the case of several groups.[2] In this paper, we associate for the first time the groups to successive time periods.[3] By initially running a separate principal component analysis on each subperiod, we observe that the factor loadings remain fairly constant across subperiods whereas the volatility of the factors fluctuate extensively through time. These results stay valid regardless of the number and the nature (non-overlapping vs. overlapping and equal size vs. unequal size) of subperiods considered. We also notice that the variance accounted for by the first factors changes substantially from subperiod to subperiod. We then propose different analyses allowing one to estimate either only permanent factors, or both permanent and transitory factors, using successively two, three, four and eight non-overlapping subperiods. We conclude that the factor structure has not changed appreciably and that permanent factors should be estimated using the common principal component approach.

Our conclusions are perfectly in line with those reached by Diebold and Li (2002) in a different setting, i.e. the Nelson-Siegel framework. In order to

[2] An application of the CPC analysis to the dynamics of interest rates, defining groups as countries in order to estimate local and global factors, can be found in Pérignon and Villa (2002).

[3] We thank Richard Roll for encouraging us to use the CPC framework in a multi-period setting.

forecast the U.S. term structure of Government bond yields, they model the yield curve with three latent factors, imposing a particular functional form on the factor loadings. Doing so the estimated factors can be interpreted as level, slope and curvature. They constrain the loadings of the first factor to be equal to one, empirically find that the loadings of the remaining two factors are constant through time, and report a significant heteroscedasticity in the dynamics of the tree factors.

The remaining of the paper is organized as follows. Section I presents an intuitive presentation of the CPC analysis. Section II describes the estimation and the model selection procedures. Section III presents an empirical analysis based on the U.S. term structure of bond yields over the last four decades. Section IV applies the CPC methodology to time-varying correlation matrices. Section V concludes the paper.

I An Intuitive Presentation of the Common Principal Component Model

The covariance matrix of bond yields is linear in a set of state variables which can be interpreted as the variances of common factors driving the term structure. These common factors are usually extracted trough principal component analysis.[4] Since a covariance matrix represents a measure of the risk associated with movements in interest rates, a measure of the proportion of risk explained by each factor can be obtained by expressing the associated eigenvalue as a fraction of the sum of the eigenvalues. This approach was pioneered by Litterman and Scheinkman (1991), where principal component analysis has been used to identify the factors underlying movements in interest rates. They determine that three factors explain the majority of movements in interest rates for various maturities, and associate them respectively, to the level, the slope and the curvature of the term structure of interest rates.[5] Recent applications of this method include, among others, Phoa (2000), Chapman and Pearson (2001), Ang and Piazzesi (2002), Piazzesi (2002), and Scherer and Avellaneda (2002).

Principal component analysis allows to estimate the factor structure of the term structure of interest rates during a given time period. This time period may be composed of successive subperiods characterized by different volatility regimes. When applying an ordinary principal component analysis to the whole sample period, by implicitly pooling different subperiods, the factor loadings are estimated using the weighted sum of the subperiod covariance matrices.[6] However pooling the covariance matrices is not appropriate unless

[4]An alternative approach is to use a factor analysis (see Knez, Litterman and Scheinkman (1994), Bliss (1997), and Lekkos (2001)).

[5]The second factor is also called in the literature *spread* or *steepness* factor, and the third one, *hump* or *butterfly spread* factor.

[6]The equality between the whole sample covariance matrix and the pooled covariance matrix is proved in Appendix 1.

all populations are assumed to have identical variability. Otherwise the period with the highest variability will determine largely the directions of the extracted components.

Empirical evidence shows that bond yield covariance matrices are unstable through time, or in other words, the hypothesis of homoscedasticity among subperiods may be rejected (see among others Engle, Ng and Rothschild, 1990, Collin-Dufresne and Goldstein, 2001, and Han, 2001). When instability is suspected, a straightforward way to succeed is to apply a single principal component analysis to each subperiod. This approach estimates a transitory factor structure for each subperiod and then fails to recover factors that are common across subperiods. Rather than applying an ordinary principal component analysis to situations for which it was not designed - simultaneous analysis in several time periods - one should use a method adapted to this particular situation, such as the ones presented in this paper. Indeed, in a CPC analysis, the eigenvectors of all the principal components are assumed to be identical in all subperiods whereas the associated eigenvalues are allowed to vary over time. A limitation of the CPC model is that it does not allow one to estimate subperiod-specific factors. All the estimated factors are assumed to be common in all subperiods. Fortunately, a generalization of the CPC analysis, called the partial CPC analysis, allows a subset of principal components to be common to all subperiods and a subset of principal components to be specific to each subperiod.

Graphically, the difference between a standard principal component analysis and a CPC analysis can be easily presented in a two-dimensional example. Consider two subperiods and two variables, x_1 and x_2, in each subperiod. Panel A in Figure 1 shows the two axes or principal components, z_1 and z_2, obtained from a standard principal component analysis run on each subperiod separately. We observe that the first principal components are not the same in the two subperiods and then, by orthogonality, the second components differ too. In each graph, the ellipse indicates the variability - the eigenvalue - associated with each principal component. Panel B in Figure 1 presents the two principal components estimated by running a CPC analysis jointly on both subperiods. We observe that, by construction, the two axes are the same in both subperiods but, according to the ellipse shapes, the variability of each principal component appears not to be the same.

< insert Figure 1 >

In order to clarify the problem of influence of subperiod with large variability, let's consider the following two subperiod covariance matrices, S_1 and S_2, with the pooled covariance matrix S_P.

$$S_1 = \begin{bmatrix} 4.875 & 1.950 \\ 1.950 & 2.625 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0.525 & 0.390 \\ 0.390 & 0.975 \end{bmatrix} \quad S_P = \begin{bmatrix} 2.700 & 1.170 \\ 1.170 & 1.800 \end{bmatrix}$$

The first principal component of S_1 is given by the eigenvector (0.866,0.500) representing an angle of 30°, since $\tan(30°) = 0.500/0.866$, with an associated eigenvalue of 6. Similarly, the first principal component of S_2 is given by the eigenvector (0.500,0.866) representing an angle of 60°, with an associated eigenvalue of 1.2. The CPC method estimates that the first common principal component is given by the eigenvector (0.707,0.707) representing an angle of 45°, which is exactly in the middle between the first principal component found individually in each group. The eigenvalues of the first CPC in each group are 5.70 and 1.14. On the other hand, the pooling method estimates that the first principal component of the pooled covariance matrix, $S_P = (S_1 + S_2)/2$, is given by the eigenvector (0.824,0.566), representing an angle of 34.5°, which is strongly influenced by the first group, with an associate eigenvalue of 3.5.

Thus, the CPC approach is less constraining and provides more robust estimates than a classical principal component analysis applied to the pooled covariance matrix. Indeed, homoscedasticity is no longer required and the direction of the extracted components are far less sensitive to periods of high volatility. The bias due to the use of standard principal component analysis could be even larger in time of financial stress. The differences reported at the factor loading level may induce substantial deviations when durations or Value-at-Risk are computed or interest-rate derivatives priced.[7]

[7]See subsection III-C for a numerical illustration of the difference between durations computed from principal components vs. from common principal components.

II Understanding Similarities among Subperiod Covariance Matrices of Bond Yields

A The General Framework

The first step in comparing two or more matrices is creating a metric or a statistic by which the comparison can be evaluated. Assuming normal populations, Anderson (1958, p. 405) has shown that a test of the null hypothesis:

$$H_{\text{Equality}} : \Sigma_n = \Sigma \text{ for all } n = 1, ..., N$$

against the general alternative one:

$$H_{\text{Unrelated}} : \text{at least one } \Sigma_n \text{ differs from the others}$$

can be conducted using a likelihood ratio-test.

To present this classical approach, let's consider N populations of size $l_1, ...,$ l_N (in this paper we refer to N subperiods) of multivariate observations X with covariance matrices Σ_n and sample covariance matrices S_n, $n = 1, ...,$ N. Each random vector is denoted $X_n = (X_{n1}, X_{n2}, ..., X_{nM})'$, where M is the number of variables (here maturities). When X_n is a sample from the M-variate normal distribution, $X_n \sim N(0, \Sigma_n)$, the statistics $l_n S_n$ are distributed independently according to the Wishart distribution with l_n degrees of freedom and covariance matrices Σ_n. The joint log-likelihood function of Σ_1,..., Σ_N given S_1,..., S_N is thus given by:

$$\ln L(\Sigma_1, ..., \Sigma_N) = C - \frac{1}{2} \sum_{n=1}^{N} l_n [\ln(\det \Sigma_n) + \mathrm{tr}(\Sigma_n^{-1} S_n)] \tag{1}$$

where C is a constant term and tr denotes the trace operator. Anderson shows that the maximum likelihood estimate of Σ_n under H_{Equality} is given by the $M \times M$ pooled covariance matrix, $S = l^{-1} \sum_{n=1}^{N} l_n S_n$, where l is the total number of observations in the N subperiods.

The log-likelihood ratio statistic for testing the hypothesis of equality versus the hypothesis of unrelated covariance matrices is:

$$T\,(\text{Equality}) = -2\ln\frac{L(S,...,S)}{L(S_1,...,S_N)} \tag{2}$$

$$= l\ln\det S-\sum_{n=1}^{N} l_n \ln\det S_n \tag{3}$$

where $L(S_1,...,S_k)$, respectively $L(S,...,S)$, is the unrestricted, respectively restricted to matrix equality, maximum of the likelihood function. The statistic $T\,(\text{Equality})$ is asymptotically χ^2 with $(N-1)(M(M-1)/2+M)$ degrees of freedom. Basically, in this case, each separate matrix is compared to the average of all the matrices. The more different each matrix is from the average, the less likely it is that the matrices are equal.

In contrast to the univariate situation, covariance matrices may share more complex relationships between one another than just being equal or unequal. For this reason, in a multi-period setting, additional levels of statistical similarity among subperiod covariance matrices have to be proposed: one permanent factor, the others being transitory; two permanent factors, the others being transitory; etc; only permanent factors. Extending the Anderson's approach, we are going to see below how the CPC and the partial CPC hypotheses can bridge the gap between the matrix equality and the matrix unrelatedness hypotheses.

B Estimating Permanent Factors Using the CPC model

In the CPC model, the subperiod covariance matrices are assumed to share common eigenvectors, but the eigenvalues can vary from subperiod to subperiod. Then, the sources of variation are assumed to be the same, but their magnitude may differ among subperiods. The presence of CPCs can be expressed formally by the assumption that there is a unique orthogonal matrix

A which jointly diagonalizes the N covariance matrices Σ_n:

$$H_{\text{CPC}} : A'\Sigma_n A = \Lambda_n, \qquad n = 1, ..., N \tag{4}$$

where A is the $M \times M$ matrix of the eigenvectors and Λ_n is the diagonal matrix of eigenvalues $(\lambda_{n1}, ..., \lambda_{nM})$ in the $n - th$ subperiod.

Here, the challenge is to estimate the A and Λ_n matrices from the sample covariance matrices S_n, $n = 1, ..., N$. If we assume that the CPC framework is valid, Σ_n can be written as $A\Lambda_n A'$, and the joint log-likelihood function given in Eq. (1) can be rewritten as:

$$\ln L(\Sigma_1, ..., \Sigma_N) = C - \frac{1}{2}\sum_{n=1}^{N} l_n[\ln\det(A\Lambda_n A') + \text{tr}((A\Lambda_n A')^{-1}S_n)] \tag{5}$$

Using the following matrix properties, $\det(AB) = \det(A)\det(B)$, $\det(A) = 1$ if A is orthogonal, and $\text{tr}(AB) = \text{tr}(BA)$, we get:

$$\ln L(\Sigma_1, ..., \Sigma_N) = C - \frac{1}{2}\sum_{n=1}^{N} l_n[\ln\det\Lambda_n + \text{tr}(\Lambda_n^{-1}A'S_nA)] \tag{6}$$

For fixed A, the above expression is maximized with respect to Λ_n when $\Lambda_n = \text{diag}(A'S_nA)$, where $\text{diag}(H)$ denotes the diagonal matrix with the same diagonal as H. Thus, at the maximum of the log-likelihood function, $\text{tr}(\Lambda_n^{-1}A'S_nA) = M$ and leads to the minimization of:

$$\sum_{n=1}^{N} l_n \ln\det\text{diag}(A'S_nA) \tag{7}$$

with respect to A, which is the same as minimizing:

$$\sum_{n=1}^{N} l_n[\ln\det\text{diag}(A'S_nA) - \ln\det(A'S_nA)] \tag{8}$$

since A has a unit determinant. This equation is precisely a measure of the global deviation from diagonality of the matrices $A'S_nA$ thanks to the

Hadamard inequality (see Flury, 1988, p. 68). This inequality states that for any positive definite symmetric matrix H, one has $\det H \geq \operatorname{detdiag}(H)$, with equality if and only if H is diagonal. As a result, for $S_1, ..., S_N$, the N positive definite sample covariance matrices of dimension $M \times M$ and $l_1, ...,$ l_N the respective population sizes, one has to minimize:[8]

$$\prod_{n=1}^{N} \left[\frac{\det \operatorname{diag}(A' S_n A)}{\det (A' S_n A)} \right]^{l_n} \tag{9}$$

with respect to the matrix A.[9] Minimizing this function can be viewed as trying to find a matrix A which diagonalizes jointly the matrices S_n, $n = 1, ...,$ N, "as much as it can". This result means that the CPC transformation can be viewed as a rotation yielding variables that are as uncorrelated as possible simultaneously in N subperiods.

C Estimating Permanent and Transitory Factors Using Partial CPC Models

The partial CPC analysis allows a subset of m $(< M)$ principal components to be common to all subperiods and a subset of $M - m$ principal components to be specific to each subperiod. The factors of the first group will be called the permanent factors and the factors of the second group will be called the transitory factors. Formally, the partial CPC model of order m, denoted pCPC(m), is defined as:

$$H_{\text{pCPC}} : A_n' \Sigma_n A_n = \Lambda_n, \qquad n = 1, ..., N \tag{10}$$

where Λ_n is, as in the CPC case, the diagonal matrix of eigenvalues (λ_{n1}, ..., λ_{nM}) in the $n - th$ subperiod. $A_n = (A^c, A^s_n)$ are orthogonal $M \times M$

[8] Fortran routines to execute this diagonalization can be found in Flury (1988, Appendix C).

[9] The fact that Eq. (9) can be obtained equivalently either by maximum likelihood estimation or directly using the Hadamard inequality, that is by definition distribution-free, serves as a justification for applying the normal maximum likelihood estimation of CPCs to nonnormal data.

matrices, where A^c of dimension $M \times m$ denotes the eigenvectors common to all subperiods, and A_n^s of dimension $M \times (M-m)$ the eigenvectors that are subperiod-specific. By orthogonality, the pCPC$(M-1)$ model implies the pCPC(M) model, i.e. the ordinary CPC. Therefore, one suitably restricts m to the range $1 \leq m \leq M-2$, meaning that a minimum dimension of $M = 3$ for the pCPC model is required.

Establishing the maximum likelihood function of a partial CPC model essentially follows the same lines as in the CPC model aside from respecting the additional orthogonality constraints of the specific factors. Recall that the presence of permanent and transitory principal components can formally be expressed by the hypothesis that the matrix A is partitioned into eigenvectors which are common and others that are specific (see Eq. (10)). The same system of equations as in the CPC analysis is obtained, however, a more intricate second equation links common and specific components, making a solution laborious to find. Luckily, an approximate solution is available, which is based on the insight that the m common components are estimated accurately by an ordinary CPC model. This assertion is proved in Flury (1988, p. 129).

D Model Selection

Choosing the relevant number of permanent and transitory factors driving the dynamics of the term structure of interest rates is a crucial step. The main advantage of the normality assumption is that likelihood-ratio tests can be derived. In fact, it is possible to test any of the hypotheses, H_{Equality}, H_{CPC}, H_{pCPC}, and $H_{\text{Unrelated}}$, against each other. For instance, the log-likelihood ratio statistic for testing the CPC hypothesis (H_{CPC}) against the unrelated covariance matrices hypothesis ($H_{\text{Unrelated}}$) is given by:

$$T\,(\text{CPC}) = \sum_{n=1}^{N} l_n \ln \det \hat{\Sigma}_n - \sum_{n=1}^{N} l_n \ln \det S_n \tag{11}$$

where $\hat{\Sigma}_n$ is the covariance matrix when the CPC hypothesis is assumed. Since the number of parameters estimated in the CPC model is $M(M-1)/2$ for the orthogonal matrix A, plus NM for the eigenvalues Λ_n, and the number of parameters in the unrelated case is given by $NM(M-1)/2 + NM$, then the statistic $T\,(\text{CPC})$ is asymptotically χ^2 with $(N-1)M(M-1)/2$ degrees of freedom.

It can be noticed however that the chi-square is not a very good fit index in practice because it is affected by both the sample and the model size. Indeed, larger samples produce larger chi-squares that are more likely to be significant (Type I error) and small samples may be too likely to accept poor models (Type II error). Moreover, more complicated models with many parameters tend to have larger chi-squares.

Instead of testing successively for the fit or lack of fit of each model, the overall best fitting model should be chosen. The best fitting model can be evaluated using a Bayesian information criterion (BIC), as the one proposed by Schwarz (1978). This criterion balances the goodness of fit of a particular model - the maximum of the log-likelihood function in this case - against the number of parameters used to fit the model, and gives a more severe complexity penalty than the Akaike information criterion (Akaike, 1987). Models with more parameters tend to fit better out of necessity, so the best model in this scheme is chosen using a "penalized log-likelihood", which is basically a simple difference between the maximum of the log-likelihood function and a multiple of the number of parameters. The BIC is defined as:

$$\begin{aligned} \text{BIC} &= -2(\text{maximum of log-likelihood}) \\ &\quad + \ln(l)(\text{number of parameters estimated}) \end{aligned} \tag{12}$$

where l denotes the total number of observations in the N subperiods. The model with the lowest BIC is the best fitting one.

III Empirical Analysis

A Data

In this section, we apply the methodology presented in the former sections to the U.S. term structure of interest rates over the last four decades. The data used in this empirical analysis are the zero-coupon bond yields from January 1960 to December 1999. According to the NBER, this forty-year sample period contains six major recessions and six major expansions.[10] Several historical and economic events occured during our period of analysis (e.g. the Vietnam war, the oil shocks, the "monetary experiments", the 1987 crash, the Gulf war) and significantly affected the bond yield curve. The bond yields are from the Fama Treasury Bill Term Structure CRSP file (1, 2, 3, 4, 5, 6 months) and the Fama-Bliss Discount Bonds CRSP file (1, 2, 3, 4, 5 years). The eleven bond yield time-series are continuously compounded and available with a monthly frequency.[11] We report in Table I that, over the last four decades the bond yields increase in average with the maturity: the term structure is upward sloping. The volatility of bond yield generally decreases with maturity. Bond yields are non-stationary, highly autocorrelated (around 0.980), and their distribution appears to be slightly leptokurtic. On the other hand, bond yield changes are stationary, far less persistent (around 0.100), but their distribution appears to be more leptokurtic. The use of non-stationary variables in a principal component analysis increases the importance of the first factor since a trend component is captured, and may lead to a spurious analysis. Although the excess kurtosis does not affect the estimates of common principal components, time dependence affects both the estimation and the testing procedure since our methodology requires

[10] The NBER peaks are 1960, 1969, 1973, 1980, 1981, 1990 and 2001, and the NBER troughs are 1961, 1970, 1975, 1980, 1982 and 1991.

[11] This database is a refinement of the one used by Fama and Bliss (1987), and is continuously updated by the *Center for Research in Security Prices* (CRSP).

i.i.d. data.[12] Furthermore, in the most recent decade, the excess kurtosis decreases and the bond yield changes become almost Gaussian. For all these reasons, we use the demeaned bond yield changes in the following empirical analysis.

< insert Table I >

B Estimation of the Risk Factor Structure

Principal Component Analysis over the Whole Sample Period

We first run a principal component analysis over the whole sample period, from January 1960 through December 1999. The first three factors explain 96.3% of the variation in bond yield changes. The first factor accounts for the most important part, 79.6%, the second one, 11.9%, and the third one, 4.8%. Figure 2 plots the factor loadings of the first three principal components: according to the shape of the three curves, and then according to how shocks to these factors affect the yield curve, these factors are labelled level, slope and curvature factors respectively. Figure 3 plots the value of the first three factors, defined as linear combinations of the original time series of bond yield changes, where the weights are given by the factor loadings. What appears clearly is the high volatility of all three factors during the 1979-1982 period, which corresponds to the period during which the Federal Reserve focused primarily on reducing the rate of growth of monetary aggregates, rather than targeting interest rates, in an effort to reduce inflation. This period is known in the literature as the Federal Reserve or the monetary experiment and took place during the Fed Chairman Volker era. This apparent structural shift is also confirmed by the values of the standard-deviations of bond-yield changes, computed using a 24-month moving window, presented in the lower panel of Figure 3.

[12] We thank Michael Rockinger for pointing out the significant effect of strong temporal dependence on our methodology.

However, running a single principal component analysis over the whole sample is valid if, and only if, the covariance matrix is constant over the considered time period. In order to check the validity of this hypothesis in the case of bond yields, we split arbitrarily our four-decade sample into respectively two, four and eight non-overlapping subperiods. Using the log-likelihood ratio statistic presented in Eq. (2), we test for the equality hypothesis against the unrelated one. As the values of the log-likelihood ratios are 522.3, 2090.0, 3065.7, with 66, 198 and 462 degrees of freedom respectively, the equality hypothesis is systematically rejected at the 1% level, with two, four and eight subperiods respectively.

< insert Figures 2 and 3 >

Estimation of Transitory Factors using Separate Principal Component Analyses

We run several separate principal component analyses using respectively two, four and eight non-overlapping subperiods. We observe that the factor loadings remain fairly constant across subperiods whereas the eigenvalues fluctuate extensively through time (see Figures 4, 5 and 6). Moreover, these results stay valid regardless of the number of subperiods considered. Table II shows that the variance accounted for by the first three factors changes from subperiod to subperiod: for example, when the whole sample period is split into eight subperiods, the variability of the original data captured by the first factor is between 59.0 and 85.6%, between 9.0 and 30.1% for the second one, and between 2.2 and 6.9% for the third one.

< insert Figures 4, 5 and 6 >
< insert Table II >

We also consider 5 and 10-year overlapping subperiods. The size of the rolling windows is maintained constant by adding a year on the front and dropping a year of the back. In this case too, the factor loadings remain fairly constant across subperiods and the eigenvalues fluctuate through time (see Figures 7 and 8).

Finally, we use a third partition suggested by the evolution of both the factors and the standard-deviations of the bond-yield changes, presented in Figure 3. We divide, as in Bliss (1997), the total sample into three subperiods of different sizes. The first period is from January 1960 through September 1979, the second from October 1979 through October 1982, and the third, from November 1982 through December 1999.[13] Notice that the second subperiod coincides with a period of extraordinary volatility of interest rates, as shown in the lower panel of Figure 3. With this alternative partition too, what stands out is the consistent pattern of the factor loadings and the high variability of the eigenvalues in the three subperiods (see Figure 9).[14]

< insert Figures 7, 8 and 9 >

Since it appears empirically that the eigenvector structure is stable but the eigenvalues change over time, and this, irrespective of the number and the nature of the considered windows, the CPC methodology is believed here to be particularly accurate.

[13] Notice that the 1973-1974 oil shock period may also have been selected as a separate subperiod, but was more limited in time.

[14] In order to test whether the small size of the second subperiods (only 37 monthly observations) poses an estimation problem, we run the same analysis extending the size of the high-volatility period (respectively to 50, 60 and 70 monthly observations). The results were not qualitatively changed.

Estimation of Permanent Factors using Common Principal Component Analyses

In this subsection, we run different analyses estimating only permanent factors (CPC) using successively two, four and eight non-overlapping subperiods of equal size and three subperiods suggested by the evolution of interest rates.[15] The CPC decomposition of the covariance matrices is presented in Eq. (4). Recall that this statistical approach allows one to estimate common factors among several subperiods without assuming that the eigenvalues and the covariance matrices are constant through time.

Figure 10 shows that the factor loadings of the first three CPCs are in the range of those obtained by running a principal component analysis on each subperiod. This result illustrates the fact that a CPC can be seen as the best compromise among all subperiod-specific principal components. The main advantages of these CPC estimates are that they are common among subperiods and that the presence of different levels of volatility is formally taken into account. On the other hand, the cost of the joint diagonalization of several covariance matrices is a slight decrease in the three-factor model performance, although still the performance remains very high.[16] Moreover, it is worthwhile to notice that the factor loadings of the first three CPCs are not substantially affected by the number of subperiods considered (i.e. two, three, four and eight subperiods). This last point highlights an advantage of the CPC approach since it appears that the only exogenous parameter in the analysis - the number of subperiods - has only a marginal impact on the estimated factors.

< insert Figure 10 >

[15] Notice that the CPC methodology does not require the different subperiods to be of equal size.

[16] For instance, in the two-subperiod case, 94.64% and 97.14% of the variability of the original data are captured by the first three principal components obtained from two separate principal component analyses; vs. 94.49% and 97.09% with the first three CPCs.

Are the Dynamics of Interest Rates Governed by Transitory, Permanent Factors, or Both?

In order to choose the number of permanent and transitory factors driving the dynamics of the term structure of interest rates, we compare the fit of alternative models based only on permanent factors (CPC), or on m permanent and $M - m$ transitory factors (pCPC(m)), $m = 1, ..., 9$. Our tests are run successively with two, four and eight subperiods of equal size and with three subperiods suggested by the evolution of interest rates. Recall that the CPC decomposition of the covariance matrices is presented in Eq. (4) and the pCPC(m) decomposition in Eq. (10). Our procedure for selecting the number of permanent factors relies on the Bayesian Information Criterion (BIC). In order to clarify the results, we report in Table III a relative BIC, defined as (min(BIC)/BIC)$\times$100.[17] By construction, the model with the lowest BIC has a relative BIC of 100. Irrespective of the number of subperiods considered, the best fitting model appears to be the CPC one, attesting that all principal components have the same financial meaning in the successive time periods. This result appears particularly striking in the case of the three-period partition since the subperiods have been chosen to be as different as possible, and each subperiod is characterized by a different volatility regime.

Two major conclusions can be drawn: First, the matrix equality hypothesis is clearly rejected since the eigenvalues appear far from being constant through time. This confirms the results of the log-likelihood ratio tests previously performed and clearly indicates that running a classical principal component analysis over the whole sample period is inaccurate. Second, as the optimal approach is the use of the CPC model, the parsimonious factor structure assumed by Engle, Ng and Rothschild (1990) and Han (2001) are validated by our tests.

[17] Notice that we use min(BIC)/BIC rather than BIC/min(BIC) because here the BICs are negative.

< insert Table III >

C Principal Component-Based Durations vs. Common Principal Component-Based Durations

To quantify the bias due to the use of classical principal components in the presence of time varying covariance structure, we are going to measure the difference between durations based alternatively on principal components and common principal components. We extend the Barber and Copper (1996) procedure for computing multi-shift durations using common principal components.[18] Let denote $r_0(s)$ the initial set of spot rates of maturity s, $r(s)$ the set of spot rates after one or several shocks, and $x(s) = r(s) - r_0(s)$ the set of spot rate changes. After a shock, modeled by a random variable h, the set of spot rates is assumed to change by an amount $u(s)h$, where $u(s)$ is a known function of the maturity date. After several shocks, modeled by random variables h_j, $j = 1, ..., J$, the set of spot rates is assumed to change by an amount $\sum_{j=1}^{J} u_j(s)h_j$. Let $P_i = \exp(-r(t_i)t_i)$ be the price of one dollar promised at date t_i. Suppose that F cash flows have to be received at dates $t_1, ..., t_F$. The discounted value of the cash flow stream $C_1, ..., C_F$ is:

$$S = \sum_{i=1}^{F} P_i C_i \tag{13}$$

The change in value of S in response to a small change in the value of the random variables h_j can be approximated by:

$$\Delta S = -\Delta h_1 \sum_{i=1}^{F} P_i C_i u_1(t_i)t_i - ... - \Delta h_J \sum_{i=1}^{F} P_i C_i u_J(t_i)t_i \tag{14}$$

$$\Delta S = -S\frac{\Delta h_1}{\sqrt{N}}D_S^1 - ... - S\frac{\Delta h_J}{\sqrt{N}}D_S^J \tag{15}$$

$$D_S^j = \frac{\sqrt{N}}{S}\sum_{i=1}^{F} P_i C_i u_j(t_i)t_i \qquad j = 1, ..., J \tag{16}$$

[18] The details of the demonstation yielding the duration formulas can be found in Appendix 2.

where D_S^j is the partial duration of S for direction j and N is the number of maturities. The computation of the D_S^j partial durations requires a proper specification for the $u_j(s)$ functions. The former functions can be obtained either by the factor loadings of the first J principal components, as advocated by Barber and Copper, or by the factor loadings of the first common principal components of the term structure.

As an illustration, we consider a bond paying \$100,000 in 1,...,6 months, and in 1,...,4 years, and \$1,000,000 at maturity, in 5 years. We assume that these cash flows are discounted using the following spot interest rates $r(t_i)$, $t_i = 1M, ..., 5Y$.[19] $u_j(t_i)$, $j = 1, 2, 3$, are the factor loadings of the first three principal components estimated over the last ten years, and $u_j^C(t_i)$, $j = 1, 2, 3$, are the factor loadings of the first three common principal components estimated over the last ten years, using two subperiod covariance matrices defined over two non-overlapping periods of five years. We compute the partial durations $D_S^j(u_j)$ and $D_S^j(u_j^C)$ of this bond in January of each year between 1970 and 1999.

For instance, the situation in January 1984 is described in Table IV. The $u_j(t_i)$ functions, $j = 1, 2, 3$, are estimated over the period January 1974 - December 1983 and the $u_j^C(t_i)$ functions, $j = 1, 2, 3$, over the following two subperiods, January 1974 - December 1978 and January 1979 - December 1983. The rates $r(t_i)$ are observed on January 1984. The partial durations of a bond paying the cash flows C presented in the last column of Table IV, are $D_S^1(u_1) = 2.489$ and $D_S^1(u_1^C) = 2.254$, $D_S^2(u_2) = 4.252$ and $D_S^2(u_2^C) = 5.236$, and $D_S^3(u_3) = -3.772$ and $D_S^3(u_3^C) = -2.315$, using the principal components and the common principal components, respectively The differences between the principal component-based durations and the common principal component-based durations are 9.4, 23.1 and 38.6%, respectively.

Figure 11 shows the partial durations estimated from the factor loadings of

[19] Notice that the spot interest rates are proxied by the bond yields.

the first three principal components and of the first three common principal components, over the period 1970-1999. Thus, for each partial duration, we get a 30 observation time-series. We observe that the two approaches can yield substantially different estimates of the durations. The reported differences are as high as 24.7% for factor 1 in January 1977, 31.3% for factor 2 in January 1997, and 57.6% for factor 3 in January 1982. We conclude that risk management techniques that ignore the time-varying nature of covariance of bond yields are likely to provide erroneous risk assessments.

$<$ **Insert Table IV** $>$
$<$ **Insert Figure 11** $>$

IV Extracting Factors from Time-Varying Correlation Matrices

Most of the empirical papers studying the comovement among financial assets focus on covariance matrices. Much less attention has been paid to the correlation structure of asset returns. Erb, Harvey and Viskanta (1994), Longin and Solnik (1995), Solnik, Boucrelle and Le Fur (1996), and Ball and Torous (2000) test the stationarity of the correlation of stock returns internationally, and conclude that correlation is not constant over time. More recently, Collin-Dufresne and Goldstein (2001) show empirically that the correlation among bond yields turn out to be also time-varying. As principal component analysis can be applied either to covariance or correlation matrices, we are going to see in this section how the common principal component methodology can be used to extract factors from time-varying correlation matrices. Notice that correlation matrices have to be used when the units of measurement are really different, or when the variability of one or several variables is very high compared to the variability of other variables. The reason being that a correlation matrix is nothing else than a covariance matrix computed from standardized data.

A convenient way to test the null hypothesis of a constant correlation matrix is to test for the equality of the correlation matrices computed over different subperiods. One of the most popular statistical test is the one developed by Jennrich (1970), based on the normalized difference between N sample correlation matrices, denoted $\Gamma_1, ..., \Gamma_N$, and defined over subperiods of respective sizes $l_1, ..., l_N$. This test is based on the following statistic:

$$\chi^2 = \sum_{n=1}^{N} \left\{ \frac{1}{2} tr\left(Z_n^2\right) - \text{diag}\left(Z_n\right)' S^{-1} \text{diag}\left(Z_n\right) \right\} \tag{17}$$

where $Z_n = \sqrt{l_n}\Gamma^{-1}\left(\Gamma_n - \Gamma\right)$, in which $\Gamma = l^{-1}\sum_{n=1}^{N} l_n \Gamma_n$ and $l = \sum_{n=1}^{N} l_n$. Furthemore, diag(X) denotes the diagonal of the square matrix X in a column form, and $S = \left(\delta_{ij} + \rho_{ij}\rho^{ij}\right)$ with $\Gamma = \left(\rho_{ij}\right)$, $\Gamma^{-1} = \left(\rho^{ij}\right)$ and $\delta_{ij} = 1$ if

$i = j$ and 0 otherwise. The Jennrich statistic has an asymptotic chi-square distribution with $(N-1)\,M(M-1)/2$ degrees of freedom, if the correlation matrix is computed for M variables. Large values of χ^2 suggest rejection of the hypothesis that all N populations have the same correlation matrix.

If the sample correlation matrices in Eq. (17) are replaced by their covariance counterpart, the first term, $\frac{1}{2}tr\left(Z_n^2\right)$, is a standard asymptotic chi-square statistic for testing the equality of the N covariance matrices and the second term, $\mathrm{diag}(Z_n)'\,S^{-1}\mathrm{diag}(Z_n)$, may thus be viewed as a correction employed when testing correlation matrices. Thus, in order to get the Jennrich statistic for the equality of N covariance matrices, the second term of the equation is omitted. In that case, the statistic has $(N-1)\,M(M+1)/2$ degrees of freedom.

Using the U.S. term structure of bond yields from January 1960 to December 1999, we test for the equality of subperiod correlation matrices using successively two, three, four and eight subperiods. The correlation matrices that we use measure the correlation among demeaned bond yield changes, that is equivalent to the covariance among standardized demeaned bond yield changes. In Table V, we present the results of the Jennrich test for the intertemporal stability of the correlation and covariance matrices. The Jennrich statistics in the second column of Panel A of Table V indicate that assumption of stability of the correlation matrix is rejected at conventional significance levels, and this, regardless of the compared time periods. The statistics reported in the second column of Panel B of Table V show that the covariance matrix is also unstable and confirm the results reported in the previous section. The Jennrich statistic is much more larger for the covariance matrices than for the correlation ones, and especially in the three-subperiod case. This finding implies that the covariances of bond yields are significantly more unstable than the correlations, that is in accordance with intuition. Indeed, the correlation among bond yields on different maturities is less likely to change suddenly. On the other hand, variances may change while correlations remain constant.

The time-varying nature of the correlation matrix should be formally taken into account when risk factors are estimated. Since the correlation structure are not stationary, running a classical principal component analysis over the whole sample appears to be inaccurate in the case of correlation matrices too. To deal with this instability problem, we run several separate principal component analyses using respectively two, three, four and eight subperiods. We observe in Figure 12 that, as with covariance matrices, the factor loadings of the first three principal components remain fairly constant across the three subperiods whereas the eigenvalues fluctuate from a subperiod to another.[20] To estimate common factors among several subperiods without assuming that the eigenvalues and the correlation matrices are constant through time, a CPC analysis can still be applied. In the case of N subperiods, we can joint diagonalize the N subperiod correlation matrices using the algorithm based on the Hadamard inequality, presented in Eq. (9). However, the estimates thus obtained may not be maximum likelihood estimates, and then the model selection procedure presented in section 3.4 may no longer be valid. In Figure 12, the factor loadings of the first three CPCs are overlaid with the ones of the first three subperiod-specific principal components. We observe that the factor loadings of the CPCs are in the range of those obtained by running a principal component analysis on each subperiod. Then, it has been shown in this section that the factor structure is stable trough time, even if the correlation structure is time-varying, and that this factor structure can still be estimated by applying a CPC decomposition to the subperiod correlation matrices.

< Insert Table V >

[20] Similar patterns are obtained when the whole sample is divided into two, four and eight subperiods, but results are not reported to save space.

< Insert Figure 12 >

V Conclusion

This paper proposes a novel methodology, based on the common principal component analysis, allowing one to estimate the factors driving the term structure of interest rates, in the presence of time-varying covariance structure. The advantages of this method are first, that, unlike classical principal component analysis, common factors can be estimated without assuming that the volatility of the factor is constant; and second, that the factor structure can be decomposed into permanent and transitory common factors. In an empirical analysis, based on the U.S. term structure, we conclude that only permanent factors are relevant for modeling the dynamics of interest rates, and that the common principal component approach is more accurate than the classical principal component one to estimate the risk factor structure. Finally, we show that the CPC methodology can also be applied to time-varying correlation matrices.

Our conclusion is important from a regulatory point of view since the recommended two methods for generating a unified set or risk measures, the Value-at-Risk and the maximum loss over a large set of scenarios for movements in the risk factors, are often computed using the results of a classical principal component analysis. Besides the risk management area, other fields of research may benefit from the framework proposed in this paper. For instance, term structure model estimation with latent variables and model calibration for pricing interest-rate derivatives appear to be natural applications of the CPC model. More generally, the framework presented in this paper may be useful every time a factor structure has to be estimated, and mainly if periods of high volatility are present in the estimation window.

Our results may also be important for the affine term structure model literature (see Duffie and Kan (1996) and Dai and Singleton (2000)). Indeed, latent factors implied by estimated affine models typically behaves like the principal components extracted from the term structure of bond yields. Our results confirm that three factors explain the dynamics of the bond yields and that the interpretation of the driving forces of the yield curve are permanent through time. Moreover, since the eigenvalues of the common principal components exhibits a stochastic pattern, the volatility of the latent factors should be allowed to vary through time.

VI Appendix

A Appendix 1

The equality between the whole sample covariance matrix and the pooled covariance matrix can be proved as follows. Let denote Σ, the covariance matrix computed over the whole sample period, Σ_n, the $n-th$ subperiod covariance matrix when the total sample is split into N subperiods, and Σ_P, the pooled covariance matrix. $X = (X_1 \quad X_2 \ \ ... \ \ X_N)$ is the $(M \times l)$ vector of data over the whole sample period, $X_n = (X_{n1} \quad X_{n2} \ \ ... \ \ X_{nM})'$ is the $(M \times l_n)$ vector of data, where l is the number of observations in the whole sample period, l_n, the number of observations in the $n-th$ subperiod, and M, the number of variables.

$$
\begin{aligned}
\Sigma &= \frac{1}{l} E(XX') \\
\Sigma &= \frac{1}{l} E\left(X_1 \quad X_2 \ \ ... \ \ X_N\right)\left(X_1 \quad X_2 \ \ ... \ \ X_N\right)' \\
\Sigma &= \frac{1}{l} E\left(X_1 \quad X_2 \ \ ... \ \ X_N\right)\begin{pmatrix} X_1' \\ X_2' \\ ... \\ X_N' \end{pmatrix} \\
\Sigma &= \frac{1}{l} \sum_{n=1}^{N} E(X_n X_n') \\
\Sigma &= \frac{l_1\Sigma_1 + l_2\Sigma_2 + ... + l_N\Sigma_N}{l} = \Sigma_P
\end{aligned}
$$

B Appendix 2

In this appendix, we propose the details of the derivation of the partial durations used in section 4.3. We start from S, the discounted value of a given cash flow stream, given in Eq. (13). The change in value of S in response to a small change in the value of the random variables h_j can be approximated by:

$$
\begin{aligned}
\Delta S &= \Delta h_1 \sum_{i=1}^{F} \frac{\partial P_i}{\partial h_1} C_i + \ldots + \Delta h_J \sum_{i=1}^{F} \frac{\partial P_i}{\partial h_J} C_i \\
&= \Delta h_1 \sum_{i=1}^{F} \left[-t_i \cdot \frac{\partial r(t_i)}{\partial h_1} P_i \right] C_i + \ldots + \Delta h_J \sum_{i=1}^{F} \left[-t_i \cdot \frac{\partial r(t_i)}{\partial h_J} P_i \right] C_i \\
&= \Delta h_1 \sum_{i=1}^{F} \left[-t_i \cdot u_1(t_i) P_i \right] C_i + \ldots + \Delta h_J \sum_{i=1}^{F} \left[-t_i \cdot u_J(t_i) P_i \right] C_i \\
&= -\Delta h_1 \sum_{i=1}^{F} P_i C_i u_1(t_i) t_i - \ldots - \Delta h_J \sum_{i=1}^{F} P_i C_i u_J(t_i) t_i
\end{aligned}
$$

In the case of a single constant shift, $u_1(t_i) = k$, we have:

$$
\begin{aligned}
\Delta S &= -\Delta h \sum_{i=1}^{F} P_i C_i k t_i \\
\frac{\Delta S}{S} &= -\Delta h k \frac{1}{S} \sum_{i=1}^{F} P_i C_i t_i \\
\frac{\Delta S}{S} &= -\Delta h k D_{FW}
\end{aligned}
$$

where D_{FW} is the Fisher-Weil duration. In the case of a single non-constant shift, we have:

$$\begin{aligned}
\Delta S &= -\Delta h \sum_{i=1}^{F} P_i C_i u_1(t_i) t_i \\
\frac{\Delta S}{S} &= -\frac{\Delta h}{\sqrt{N}} (\frac{\sqrt{N}}{S} \sum_{i=1}^{F} P_i C_i u_1(t_i) t_i) \\
\frac{\Delta S}{S} &= -\frac{\Delta h}{\sqrt{N}} D_S
\end{aligned}$$

where D_S is the duration of the surplus value and N is the number of maturities. The purpose of the $\sqrt{N}$ term is to obtain the Fisher-Weil duration in the case of $u_1(t_i)$ is equal to a constant. In the case of several non-constant shifts, we have:

$$\begin{aligned}
\Delta S &= -\Delta h_1 \sum_{i=1}^{F} P_i C_i u_1(t_i) t_i - ... - \Delta h_J \sum_{i=1}^{F} P_i C_i u_J(t_i) t_i \\
\frac{\Delta S}{S} &= -\frac{\Delta h_1}{\sqrt{N}} (\frac{\sqrt{N}}{S} \sum_{i=1}^{F} P_i C_i u_1(t_i) t_i) - ... - \frac{\Delta h_J}{\sqrt{N}} (\frac{\sqrt{N}}{S} \sum_{i=1}^{F} P_i C_i u_J(t_i) t_i) \\
\frac{\Delta S}{S} &= -\frac{\Delta h_1}{\sqrt{N}} D_S^1 - ... - \frac{\Delta h_J}{\sqrt{N}} D_S^J
\end{aligned}$$

where D_S^j is the partial duration of the surplus value for direction j.

$$D_S^j = \frac{\sqrt{N}}{S} \sum_{i=1}^{F} P_i C_i u_j(t_i) t_i \qquad j = 1, ..., J$$

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VII Tables and Figures

Table I: Descriptive Statistics (January 1960 - December 1999)

	Bond Yields							
	Mean	Std-dev	Skew	Kurt	Skew 90	Kurt 90	Rho(1)	ADF
1M	5.65	2.58	1.27	2.04	0.45	0.30	0.958	-2.39
2M	5.92	2.65	1.26	1.91	0.50	0.33	0.976	-2.30
3M	6.08	2.70	1.25	1.79	0.47	0.31	0.978	-2.30
4M	6.14	2.71	1.25	1.76	0.47	0.29	0.978	-2.28
5M	6.24	2.71	1.22	1.63	0.44	0.24	0.978	-2.19
6M	6.30	2.71	1.22	1.60	0.42	0.22	0.979	-2.22
1Y	6.52	2.64	1.08	1.14	0.36	0.05	0.978	-2.06
2Y	6.72	2.59	0.99	0.85	0.46	-0.07	0.983	-1.96
3Y	6.88	2.53	0.97	0.74	0.53	-0.16	0.984	-1.92
4Y	7.00	2.50	0.92	0.63	0.51	-0.31	0.985	-1.81
5Y	7.06	2.48	0.87	0.47	0.50	-0.42	0.987	-1.75
	Bond Yield Changes							
	Mean§	Std-dev	Skew	Kurt	Skew 90	Kurt 90	Rho(1)	ADF
1M	1.74	0.74	-1.04	11.98	-0.07	3.02	-0.110	-12.2*
2M	1.87	0.57	-2.03	14.76	-0.36	1.62	0.130	-10.5*
3M	1.85	0.55	-1.43	11.83	-0.28	0.94	0.122	-10.3*
4M	1.59	0.56	-1.92	16.19	-0.18	0.21	0.131	-10.3*
5M	1.60	0.56	-1.68	15.09	-0.04	0.90	0.118	-10.3*
6M	1.28	0.54	-1.58	14.94	0.02	0.17	0.114	-10.6*
1Y	2.15	0.54	-1.07	13.27	0.25	0.26	0.114	-10.0*
2Y	2.55	0.47	-0.72	9.61	0.09	-0.40	0.159	-10.2*
3Y	2.56	0.43	-0.14	6.72	0.15	-0.36	0.120	-10.3*
4Y	2.95	0.42	-0.18	4.50	0.13	-0.30	0.066	-10.1*
5Y	3.28	0.39	-0.29	4.32	0.06	-0.22	0.082	-10.1*

Note: Descriptive statistics are computed from the 480 monthly observations from 1960:01-1999:12, for the level of the bond yields and for the bond yield changes, with eleven different maturities from one month (1M) to five years (5Y). Std-dev stands for standard-deviation, Skew for skewness, Kurt for kurtosis, Skew 90 for skewness computed over the sample 1990:01-1999:12, Kurt 90 for kurtosis computed over the sample 1990:01-1999:12, Rho(1) for first-order autocorrelation coefficient, and ADF for Augmented Dickey-Fuller test with an intercept. § indicates that the mean has been multiplied by 1000, and * that the unit root hypothesis can be rejected at the 1% level, and then that the series is stationary.

Table II: Variance Accounted for by the First Five Factors (in %)

Panel A: One, Two, Three and Four Subperiods

Factor	$\Sigma_{1/1}$	$\Sigma_{1/2}$	$\Sigma_{2/2}$	$\Sigma^u_{1/3}$	$\Sigma^u_{2/3}$	$\Sigma^u_{3/3}$	$\Sigma_{1/4}$	$\Sigma_{2/4}$	$\Sigma_{3/4}$	$\Sigma_{4/4}$
1	79.60	80.09	79.40	78.94	87.86	62.71	70.02	82.97	81.36	66.85
2	11.94	10.97	13.01	11.88	7.34	25.77	14.94	9.98	12.02	21.91
3	4.80	3.58	4.75	3.46	3.17	6.69	6.02	2.87	3.98	7.06
4	1.11	1.65	1.05	1.75	0.51	2.10	2.14	1.56	0.95	1.92
5	0.81	1.19	0.52	1.27	0.35	0.64	1.19	0.76	0.49	0.68
Cum3	96.34	94.64	97.14	94.28	98.37	95.17	90.98	95.82	97.36	95.82
Cum5	98.26	97.48	98.72	97.30	99.23	97.90	95.95	98.14	98.80	98.42

Panel B: Eight Subperiods

Factor	$\Sigma_{1/8}$	$\Sigma_{2/8}$	$\Sigma_{3/8}$	$\Sigma_{4/8}$	$\Sigma_{5/8}$	$\Sigma_{6/8}$	$\Sigma_{7/8}$	$\Sigma_{8/8}$
1	70.96	71.00	83.30	84.02	85.62	58.96	71.58	61.84
2	16.08	12.85	10.00	8.97	9.19	30.10	18.58	27.22
3	5.69	6.85	2.22	4.33	3.20	6.06	6.83	5.65
4	1.99	3.45	1.82	0.72	0.63	2.11	1.38	2.22
5	1.48	2.48	0.76	0.61	0.43	1.04	0.51	0.92
Cum3	92.72	90.70	95.53	97.31	98.00	95.13	96.99	94.71
Cum5	96.18	96.63	98.11	98.64	99.07	98.27	98.88	97.85

Note: In this table, the variance, measured in percent, accounted by the first five factors is presented. Cum3 and Cum5 indicate the proportion of the total variance of the original data captured by the first three, respectively five, factors. $\Sigma_{n/N}$ denotes the n–th subperiod covariance matrix when the total sample is split into N subperiods (n = 1, ..., N and N = 1, 2, 3, 4, 8). The subscript u means that the subperiods are of different sizes. Similar patterns are obtained with overlapping subperiods but results are not reported to save space.

Table III: Model Selection

Panel A: Two Subperiods

Model	Max(ln L)	Parameters	RBIC
Equality	8992.60	66	101.49
CPC	9157.53	77	[100]
pCPC(5)	9170.81	92	100.37
pCPC(4)	9184.09	98	100.43
pCPC(3)	9188.79	105	100.62
pCPC(2)	9205.92	113	100.71
pCPC(1)	9244.71	122	100.58
Unrelated	9253.73	132	100.83

Panel B: Three Subperiods

Model	Max(ln L)	Parameters	RBIC
Equality	8989.98	66	107.06
CPC	9678.23	88	[100]
pCPC(5)	9711.82	118	100.63
pCPC(4)	9728.10	130	100.86
pCPC(3)	9736.18	144	101.24
pCPC(2)	9769.88	160	101.41
pCPC(1)	9822.05	178	101.45
Unrelated	9865.41	198	101.65

Table III: Model Selection (Continued)

Panel C: Four Subperiods

Model	Max(ln L)	Parameters	RBIC
Equality	8955.29	66	107.99
CPC	9756.15	99	[100]
pCPC(5)	9816.80	144	100.84
pCPC(4)	9837.00	162	101.22
pCPC(3)	9849.76	183	101.78
pCPC(2)	9887.36	207	102.19
pCPC(1)	9951.64	234	102.40
Unrelated	10000.29	264	102.89

Panel D: Eight Subperiods

Model	Max(ln L)	Parameters	RBIC
Equality	8885.38	66	109.61
CPC	9957.46	143	[100]
pCPC(5)	10064.72	248	102.33
pCPC(4)	10102.79	290	103.35
pCPC(3)	10137.58	339	104.67
pCPC(2)	10223.19	395	105.69
pCPC(1)	10318.37	458	106.87
Unrelated	10418.21	528	108.28

Note: In this table, the fit of alternative models is compared; starting from Equality of the covariance matrices, then common principal component (CPC), partial common principal component of order five (pCPC(5)), etc, of order one (pCPC(1)), and ending with Unrelated covariances matrices. Max(ln L) denotes the maximum of the log-likelihood function, Parameters, the number of parameters estimated in each model, and RBIC, the relative Schwarz information criterion defined as (min(BIC)/BIC)$\times$100. By construction, the model with the lowest BIC has a RBIC of 100. For the sake of brevity, the results for pCPC(m), m = 6,7,8,9, are not reported.

Table IV: Factor Loadings, Interest Rates and Bond Cash Flows on January 1984

t_i	$u_1(t_i)$	$u_2(t_i)$	$u_3(t_i)$	$u_1^C(t_i)$	$u_2^C(t_i)$	$u_3^C(t_i)$	$r(t_i)$	P_i	C
1M	.361	-.658	-.581	.357	-.441	-.758	9.02	.993	100000
2M	.343	-.304	.119	.350	-.309	.001	8.95	.985	100000
3M	.342	-.096	.309	.354	-.195	.261	9.09	.978	100000
4M	.355	-.024	.377	.358	-.098	.322	9.13	.970	100000
5M	.349	.056	.285	.356	-.024	.281	9.21	.962	100000
6M	.344	.109	.228	.348	.027	.269	9.29	.955	100000
1Y	.321	.226	-.174	.320	.237	-.069	9.54	.909	100000
2Y	.249	.317	-.175	.233	.372	-.082	10.34	.813	100000
3Y	.211	.297	-.252	.200	.392	-.151	10.60	.728	100000
4Y	.181	.356	-.293	.173	.431	-.206	10.94	.646	100000
5Y	.166	.289	-.258	.150	.356	-.159	11.16	.572	1000000

Note: In this table, t_i denotes the maturity dates, $u_1(t_i)$, $u_2(t_i)$ and $u_3(t_i)$ the factor loadings of the first three principal components, $u_1^C(t_i)$, $u_2^C(t_i)$ and $u_3^C(t_i)$ the factor loadings of the first three common principal components, $r(t_i)$ the spot interest rates, P_i the price of one dollar promised at date t_i, and C the bond cash flows.

Table V: Test for the Intertemporal Stability of Correlation and Covariance Matrices

Panel A: Correlation Matrices

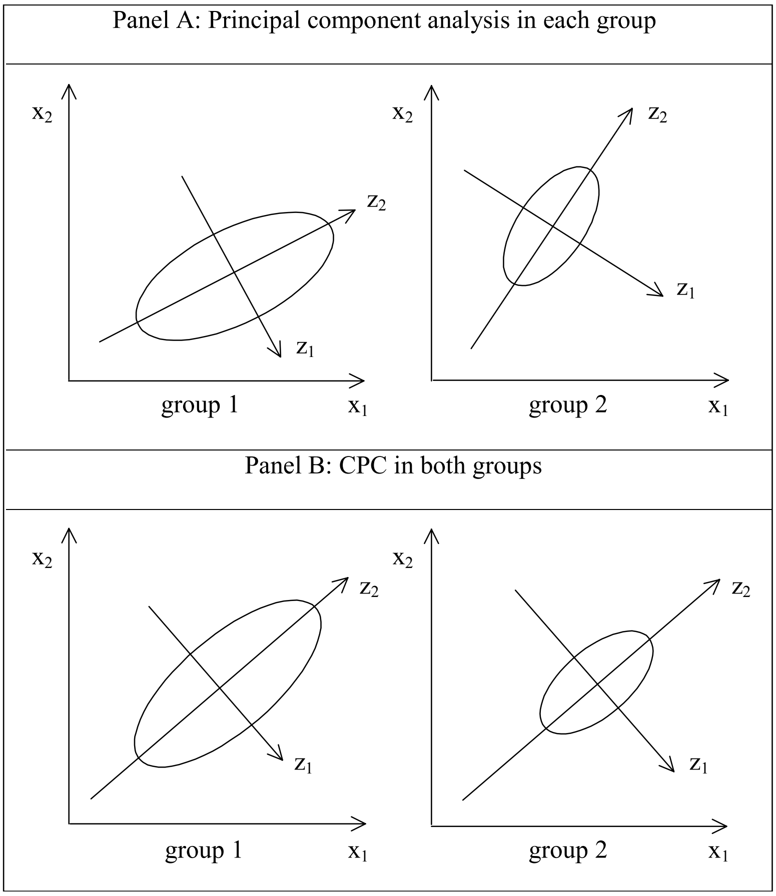
Subperiods	Jennrich χ^2	d.f.	p-value	$\chi^2_{95\%}$	$\chi^2_{99\%}$
Two	241.74	55	0.000	73.31	82.29
Three	321.65	110	0.000	135.48	147.41
Four	506.26	165	0.000	195.97	210.18
Eight	1169.00	385	0.000	431.75	452.48

Panel B: Covariance Matrices

Subperiods	Jennrich χ^2	d.f.	p-value	$\chi^2_{95\%}$	$\chi^2_{99\%}$
Two	432.64	66	0.000	85.96	95.63
Three	4056.80	132	0.000	159.81	172.71
Four	1809.00	198	0.000	231.83	247.21
Eight	3636.80	462	0.000	513.11	535.64

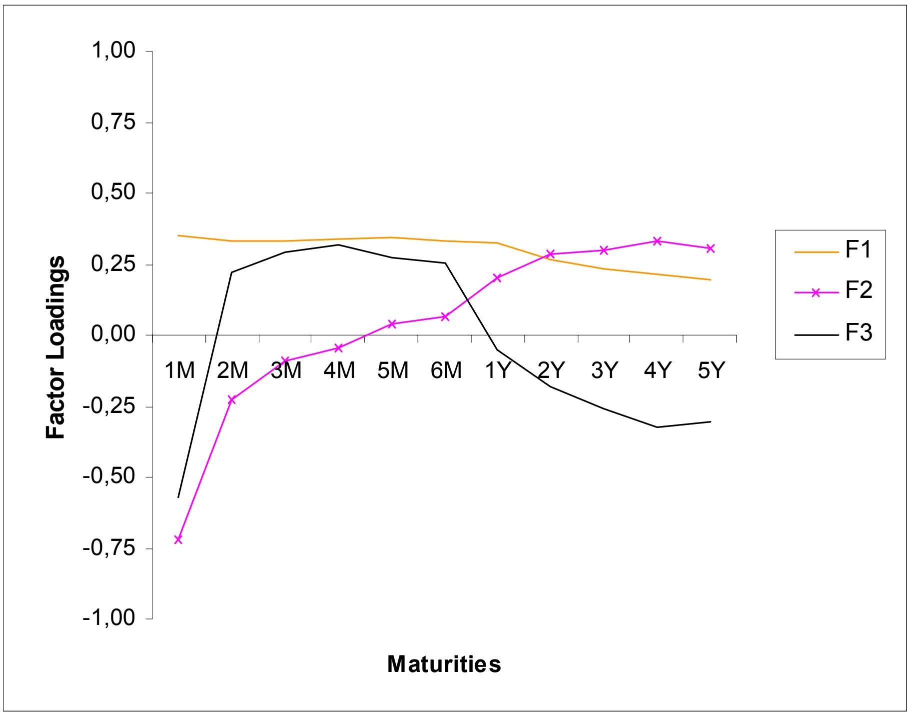
Note: In this table, Jennrich chi-square (χ^2) statistics for testing the equality of correlation (Panel A) and covariance (Panel B) matrices are reported. We test for the equality of successively two, three, four and eight subperiod correlation and covariance matrices. $\chi^2_{95\%}$ and $\chi^2_{99\%}$ denote the critical values of the chi-square distribution, with the corresponding number of degrees of freedom (d.f.), respectively at the 95% and 99% confidence levels.

Figure 1: **Comparison of Principal Components and Common Principal Components (CPC)**



Note: In this figure, we consider two groups and two variables, x_1 and x_2, in each group. Panel A shows the two axes or principal components, z_1 and z_2, obtained from a standard principal component analysis run for both groups separately. We observe that the first principal components are not the same in the two groups and then, by orthogonality, the second components differ too. In each graph, the ellipse indicates the variability (the eigenvalue) associated with each principal component. Panel B presents the two principal components estimated by running a CPC analysis jointly on both groups. We observe that, by construction, the two axes are the same in both groups but, according to the ellipse shapes, the variability of each principal component appears not to be the same.

Figure 2: **Factor Loadings of the First Three Principal Components (January 1960 - December 1999)**



Note: In this figure, the factor loadings of the first three principal components are presented. They have been estimated by applying a principal component analysis to the covariance matrix estimated over the period 1960:01-1999:12. Notice the non uniform time scale on the horizontal axis.

Figure 3: **Factor Values and Standard-Deviations of Bond Yield Changes (January 1960 - December 1999)**

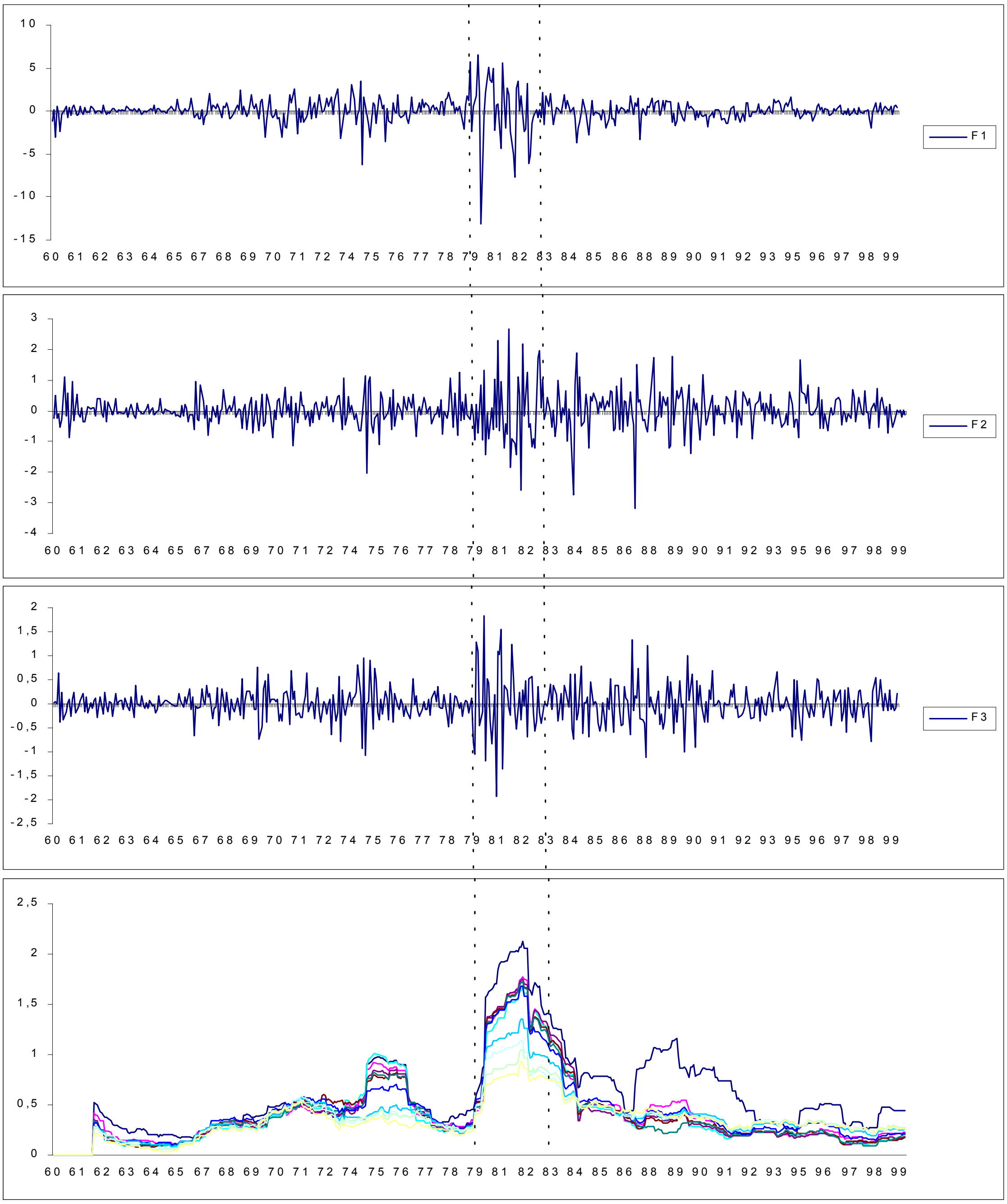


Figure 4: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into Two Subsamples**

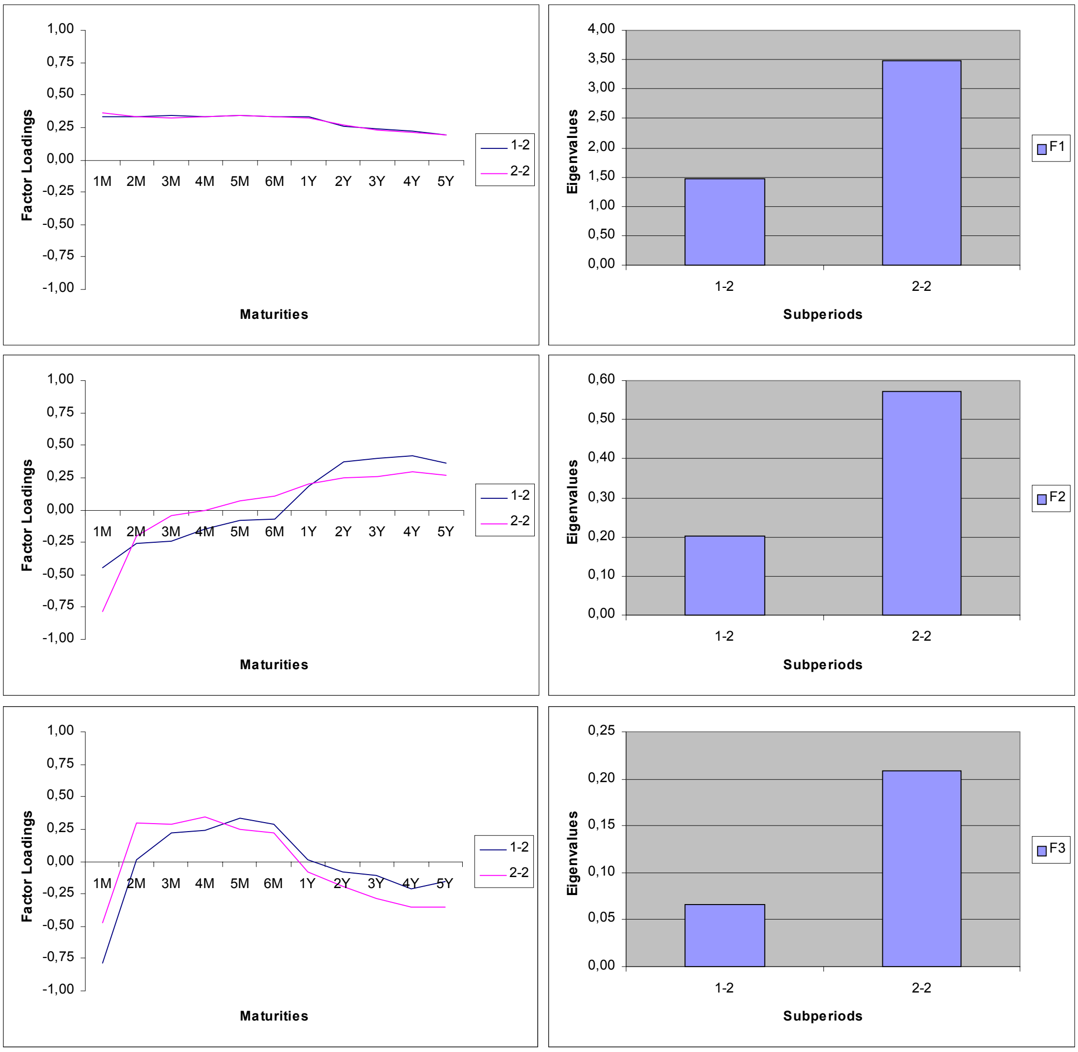


Figure 5: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into Four Subsamples**

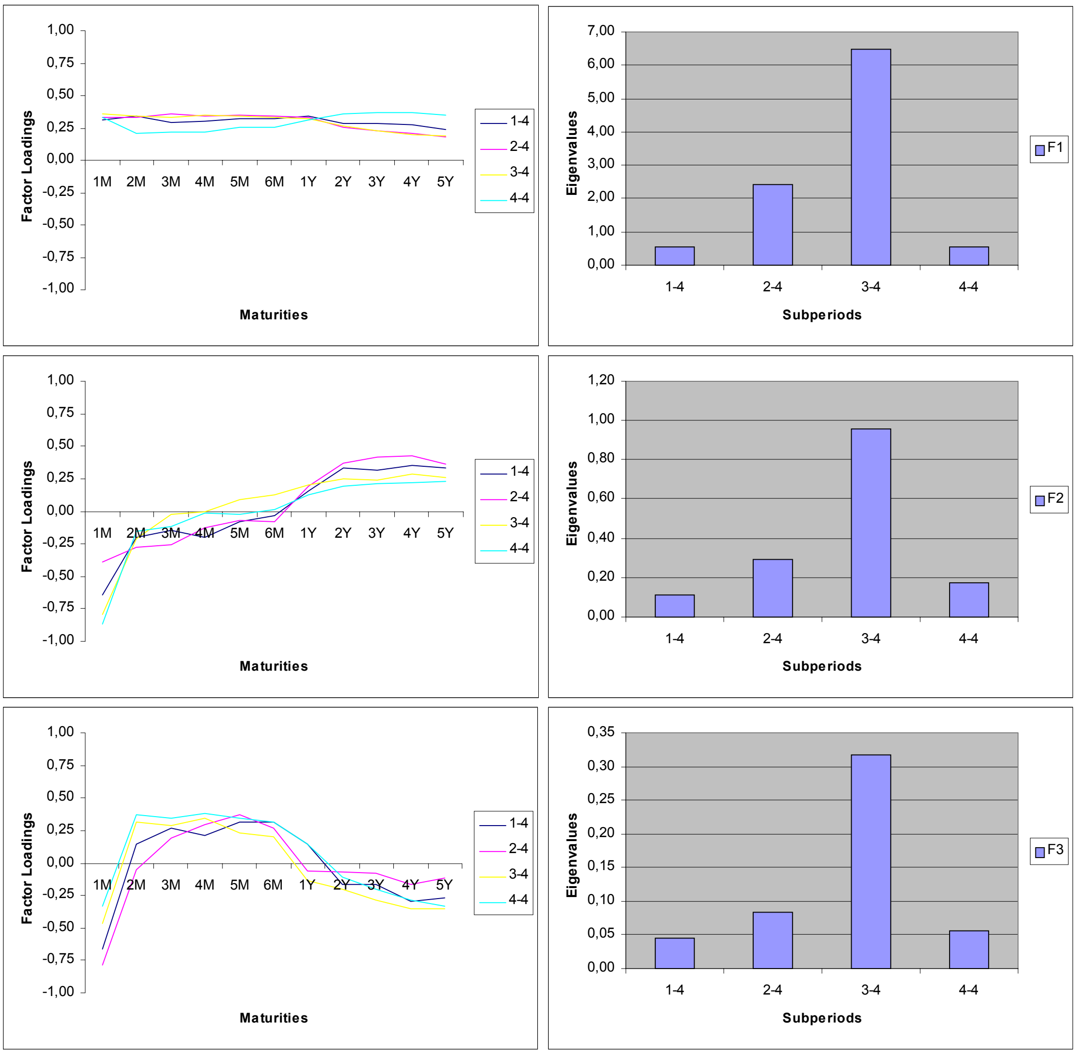


Figure 6: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into Eight Subsamples**

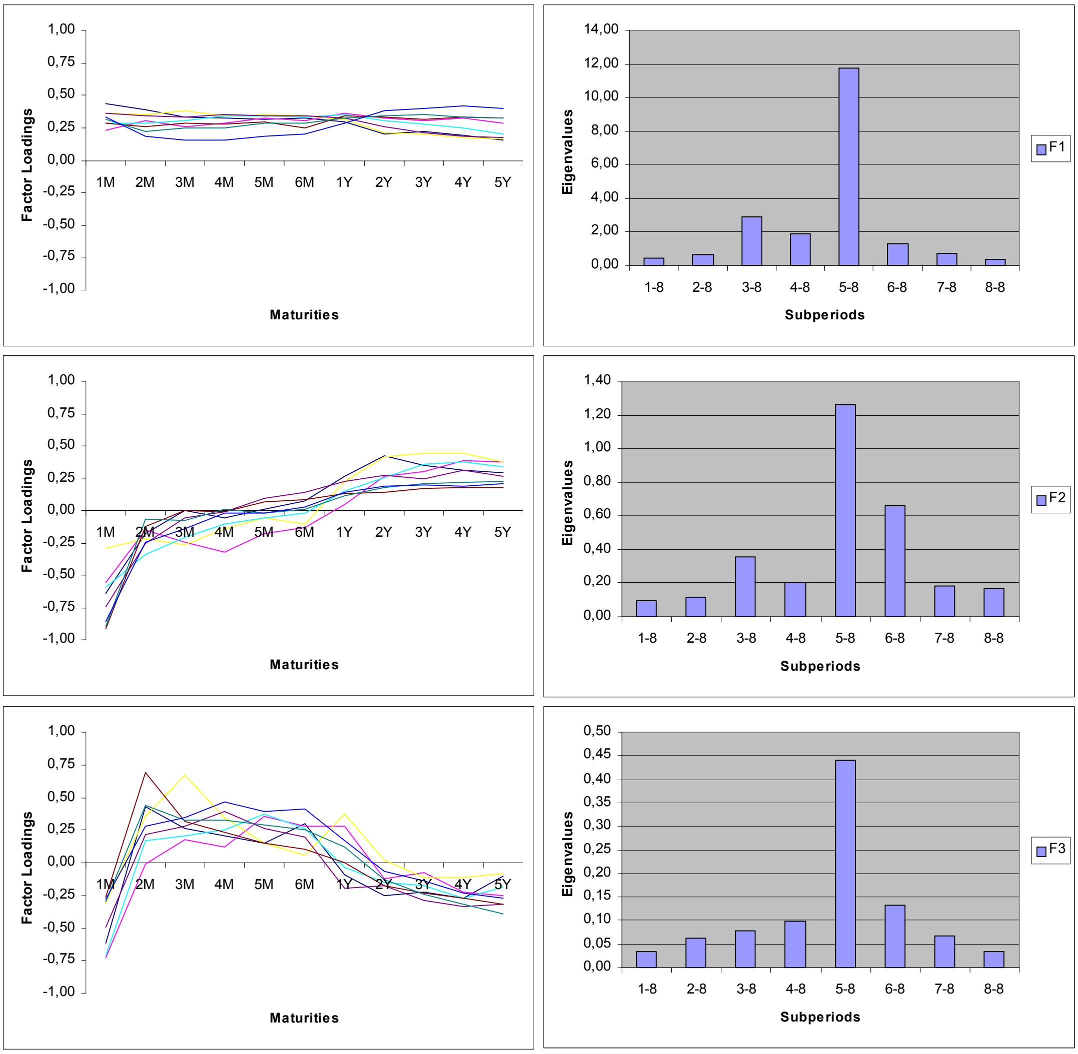


Figure 7: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into 10-Year Overlapping Windows**

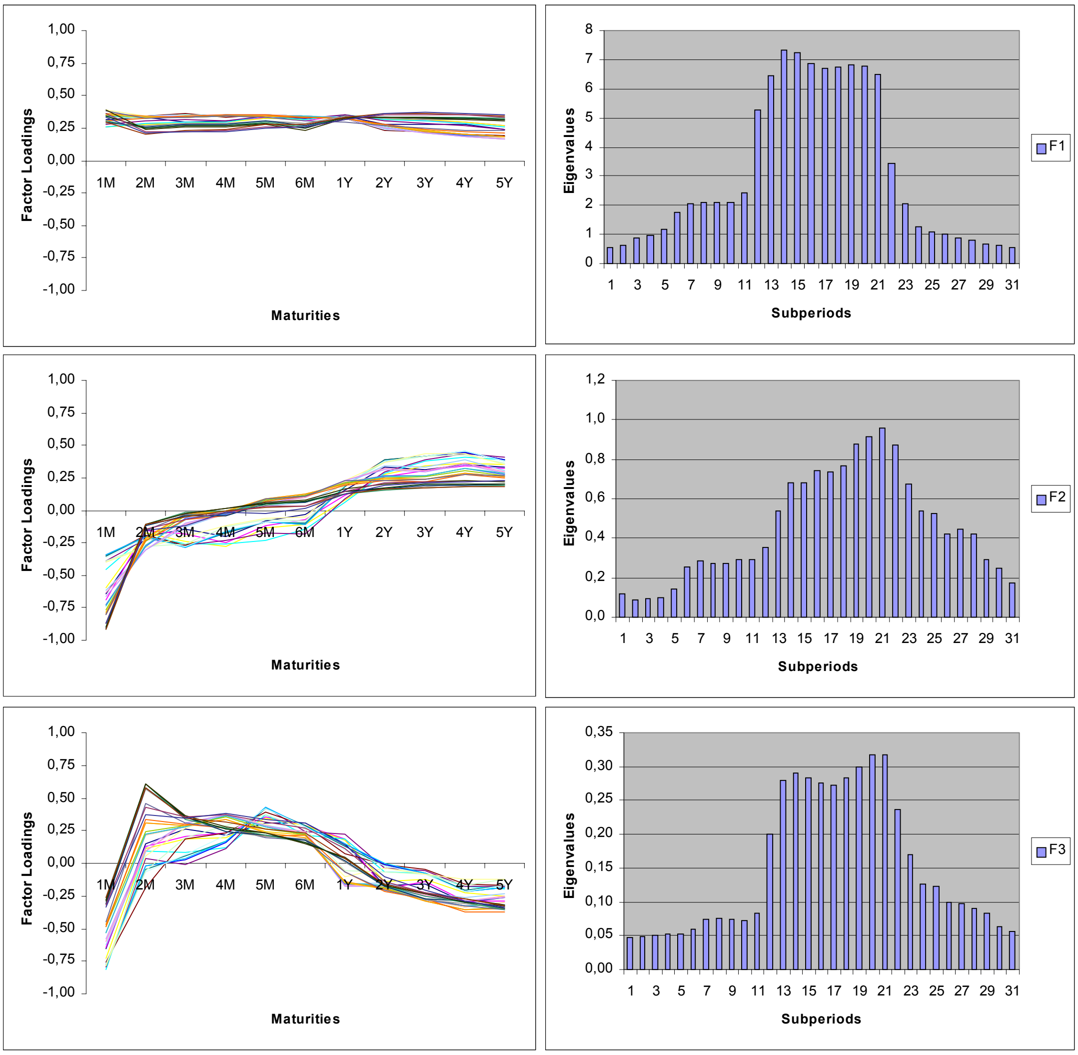


Figure 8: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into 5-Year Overlapping Windows**

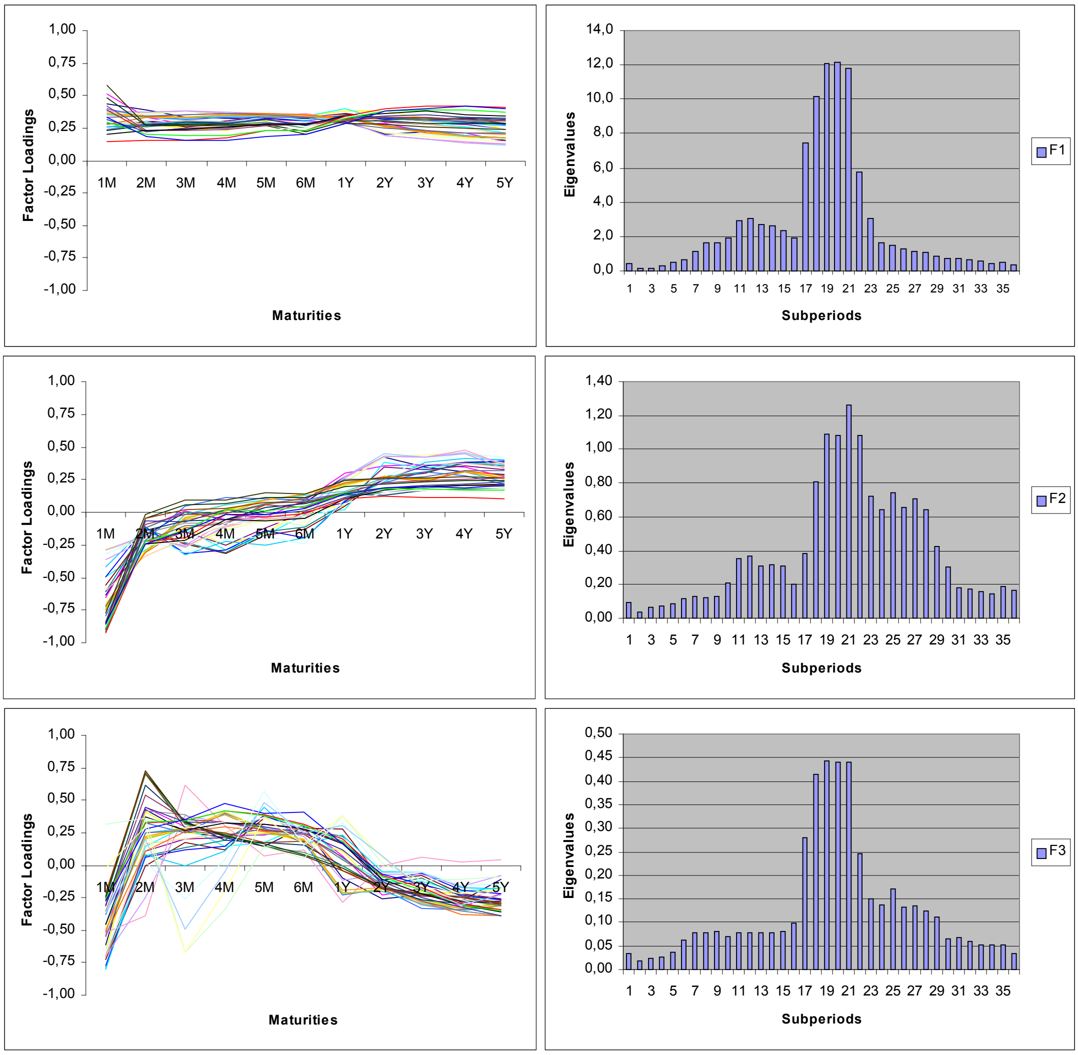


Figure 9: **Factor Loadings and Eigenvalues of the First Three Principal Components Splitting the Sample into Three Subsamples of Different Sizes**

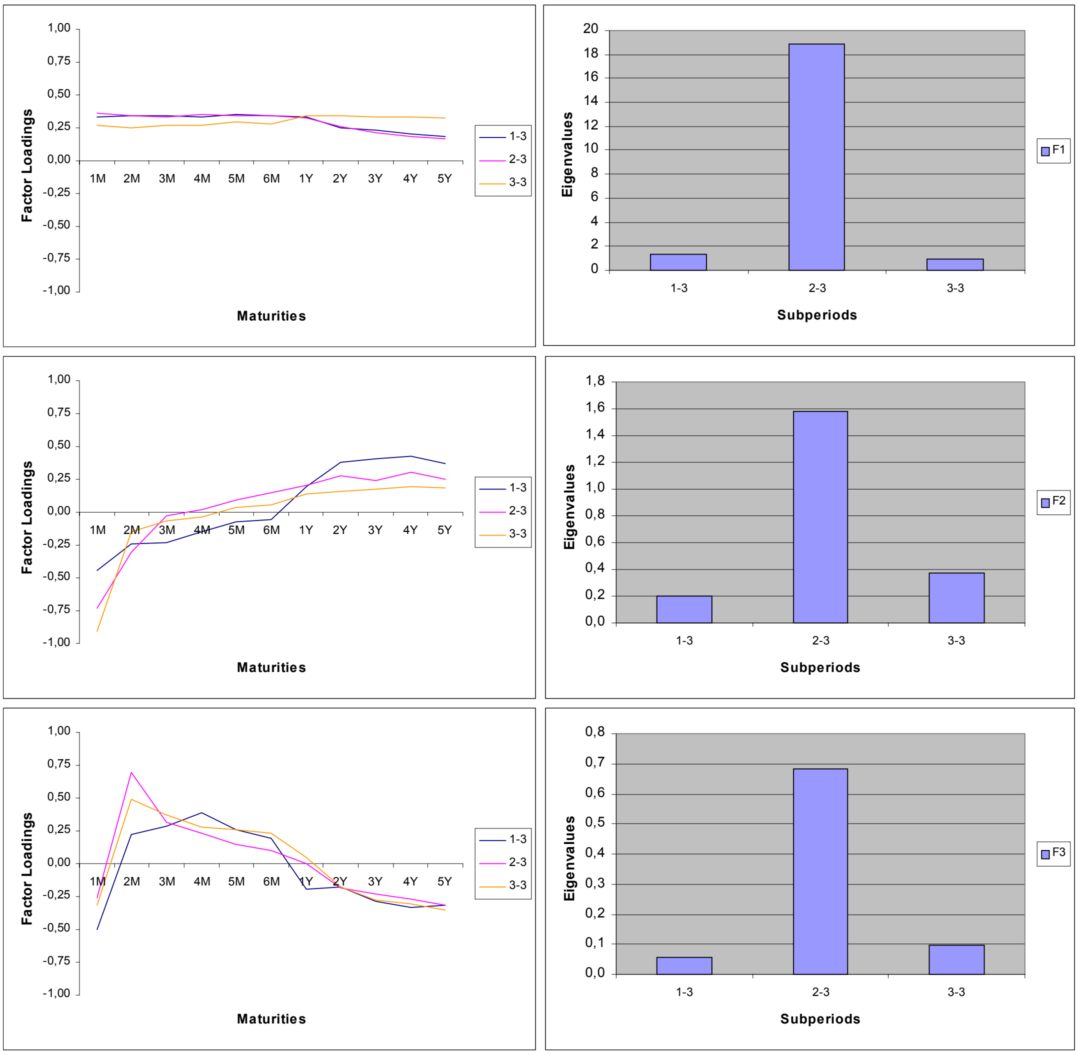


Figure 10: **Factor Loadings of the First Three Common Principal Components Estimated by Dividing the Sample into 2, 3, 4 and 8 Periods**

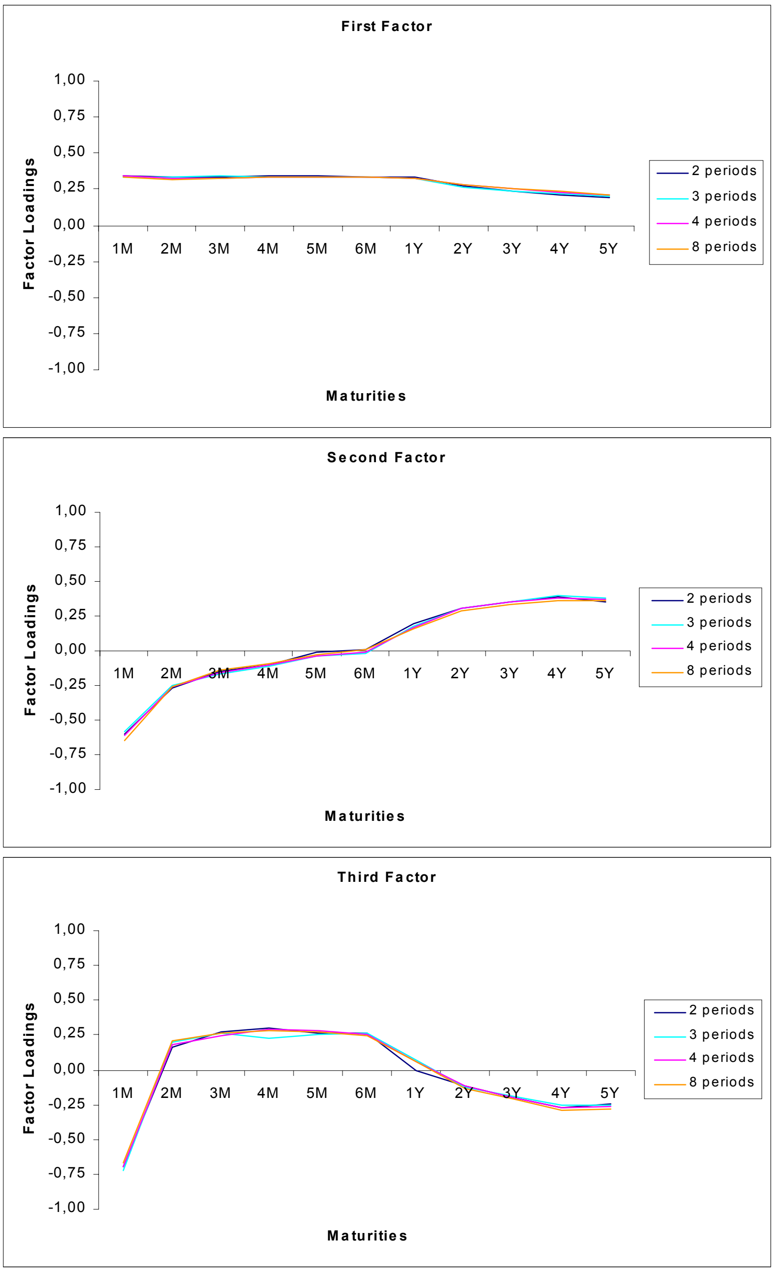


Figure 11: **Principal Component Based-Durations (DPC) vs. Common Principal Component Based-Durations (DCPC)**

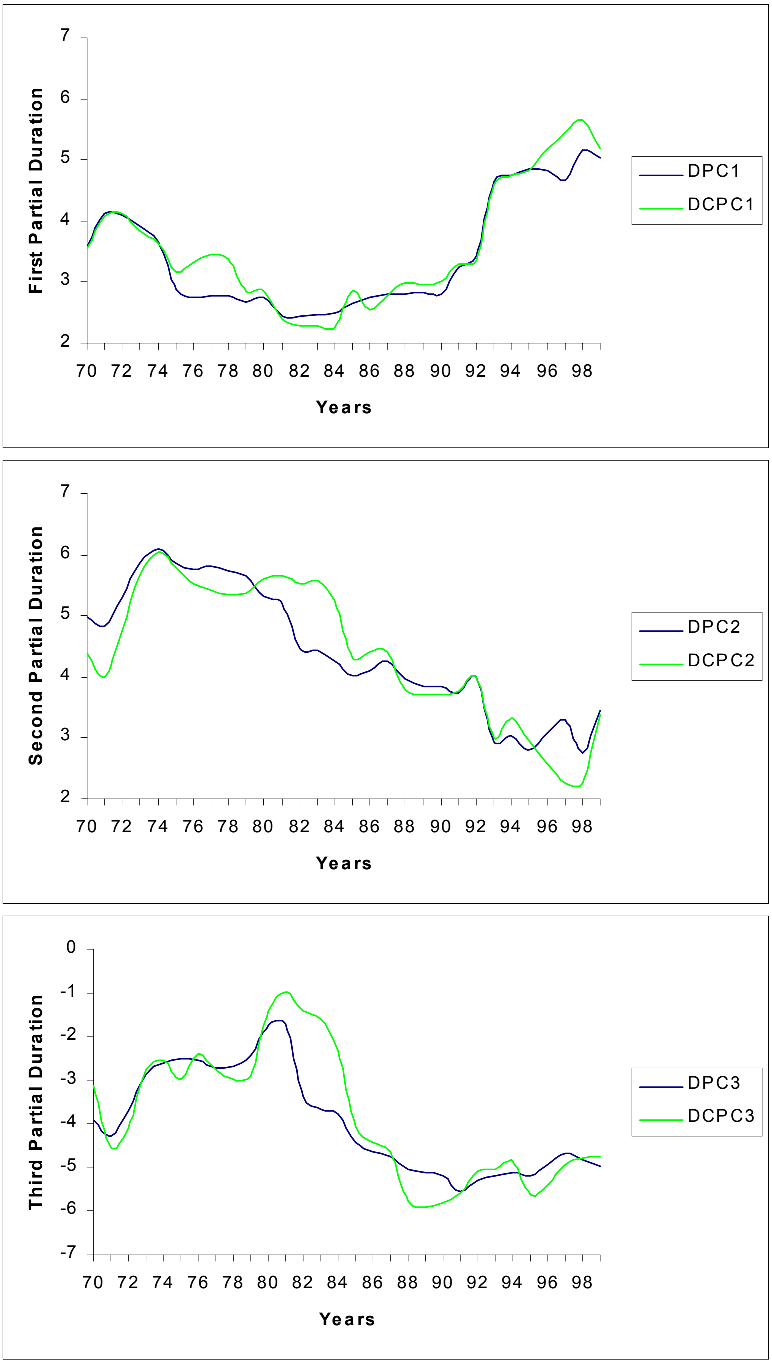
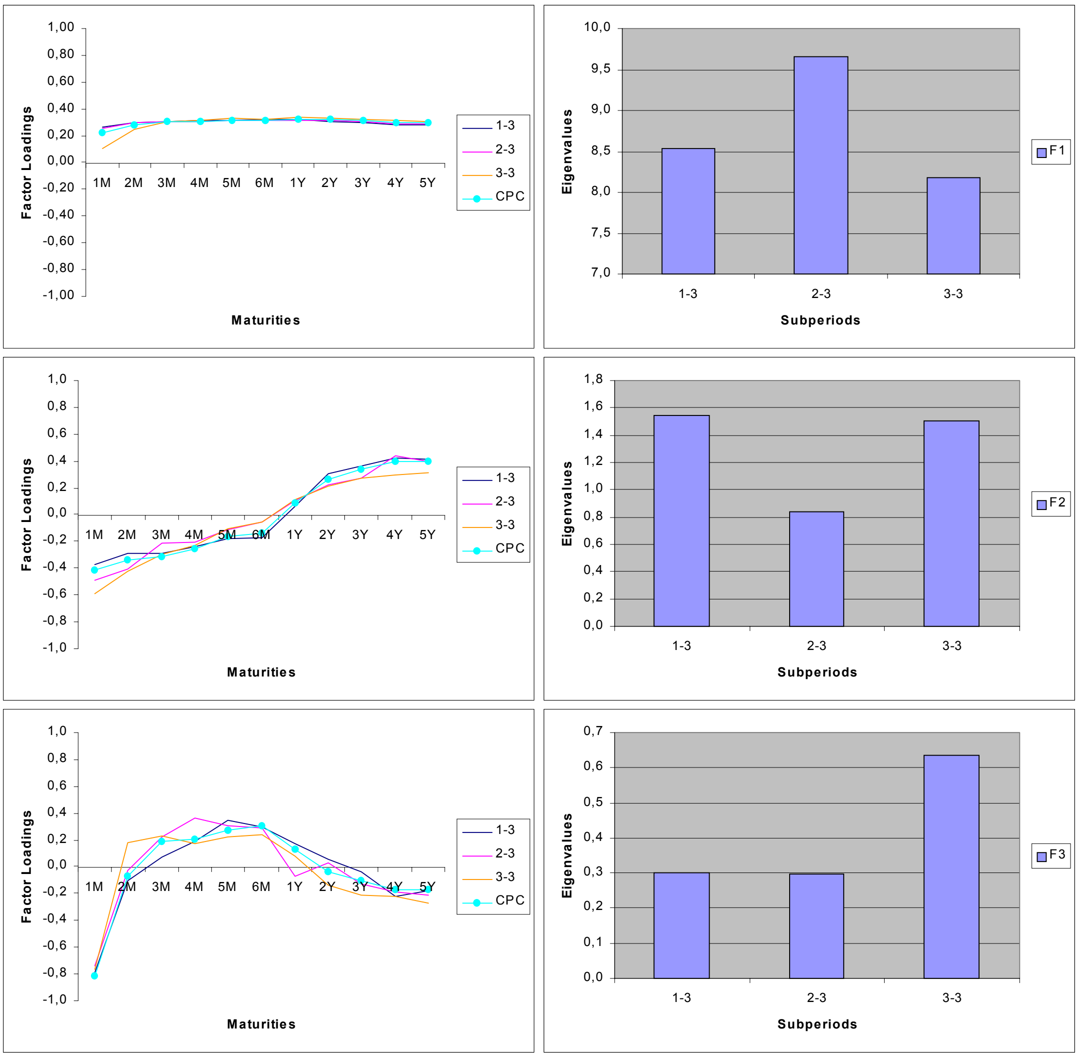


Figure 12: **Factor Loadings and Eigenvalues of the First Three Principal Components Estimated from Three Subperiod Correlation Matrices**



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More than 13’000 students, the majority being foreigners, are enrolled in the various programs from the licence to high-level doctorates. A staff of more than 2’500 persons (professors, lecturers and assistants) is dedicated to the transmission and advancement of scientific knowledge through teaching as well as fundamental and applied research. The University of Geneva has been able to preserve the ancient European tradition of an academic community located in the heart of the city. This favors not only interaction between students, but also their integration in the population and in their participation of the particularly rich artistic and cultural life. ***http://www.unige.ch***

The University of Lausanne

Founded as an academy in 1537, the University of Lausanne (UNIL) is a modern institution of higher education and advanced research. Together with the neighboring Federal Polytechnic Institute of Lausanne, it comprises vast facilities and extends its influence beyond the city and the canton into regional, national, and international spheres.

Lausanne is a comprehensive university composed of seven Schools and Faculties: religious studies; law; arts; social and political sciences; business; science and medicine. With its 9’000 students, it is a medium-sized institution able to foster contact between students and professors as well as to encourage interdisciplinary work. The five humanities faculties and the science faculty are situated on the shores of Lake Leman in the Dorigny plains, a magnificent area of forest and fields that may have inspired the landscape depicted in Brueghel the Elder's masterpiece, the Harvesters. The institutes and various centers of the School of Medicine are grouped around the hospitals in the center of Lausanne. The Institute of Biochemistry is located in Epalinges, in the northern hills overlooking the city. ***http://www.unil.ch***

The Graduate Institute of International Studies

The Graduate Institute of International Studies is a teaching and research institution devoted to the study of international relations at the graduate level. It was founded in 1927 by Professor William Rappard to contribute through scholarships to the experience of international co-operation which the establishment of the League of Nations in Geneva represented at that time. The Institute is a self-governing foundation closely connected with, but independent of, the University of Geneva.

The Institute attempts to be both international and pluridisciplinary. The subjects in its curriculum, the composition of its teaching staff and the diversity of origin of its student body, confer upon it its international character. Professors teaching at the Institute come from all regions of the world, and the approximately 650 students arrive from some 60 different countries. Its international character is further emphasized by the use of both English and French as working languages. Its pluralistic approach - which draws upon the methods of economics, history, law, and political science -reflects its aim to provide a broad approach and in-depth understanding of international relations in general. ***http://heiwww.unige.ch***

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